lektion7

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1 Lektion 7

[1]:
```
from sympy import *
init_printing()
import matplotlib.pyplot as plt
import numpy as np
#%matplotlib notebook
%matplotlib inline
%config InlineBackend.figure_format = 'svg'
```

1.1 Wiederholung

1.1.1 Sympy Ausdrücke (expressions) vs. Funktionen

[2]:
```
x = symbols('x')
f_ausdruck = x**2-sin(pi*x)
f_ausdruck
```

[2]: $x^2 - \sin(\pi x)$

[3]:
```
y = symbols('y')
f_symfunk = Lambda(y, y**2-sin(pi*y))
f_symfunk
```

[3]: $\left(y \mapsto y^2 - \sin(\pi y)\right)$

[4]:
```
f_symfunk(x)
```

[4]: $x^2 - \sin(\pi x)$

Auswerten/Einsetzen

[5]:
```
f_symfunk(2)
```

[5]: 4

[6]:
```
f_ausdruck(2)
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
/tmp/ipykernel_925273/735444820.py in <module>
----> 1 f_ausdruck(2)

TypeError: 'Add' object is not callable
```

[7]:
```
f_ausdruck.subs(x, 2)
```

[7]: 4

[9]:
```
f_symfunk.subs(2, 33)
```

[9]: $\left(y \mapsto y^{33} - \sin(\pi y)\right)$

[10]:
```
f_pyfun = lambda zahl : zahl**2 - sin(pi*zahl)
f_pyfun
```

[10]: `<function __main__.<lambda>(zahl)>`

[11]:
```
f_pyfun(2)
```

[11]: 4

[12]:
```
def f_pyfun2(argument):
    return argument**2 - sin(pi*argument)
```

[13]:
```
neuername = f_pyfun2
```

[14]:
```
neuername(2)
```

[14]: 4

[15]:
```
f_pyfun2(2)
```

[15]: 4

[16]:
```
f_ausdruck.diff(x)
```

[16]: $2x - \pi \cos(\pi x)$

[17]:
```
f_symfunk.diff(y)
```

[17]: 0

[18]:
```
f_pyfun(x).diff(x)
```

[18]: $2x - \pi \cos(\pi x)$

[19]:
```
f_pyfun2(x).diff(x)
```

[19]: $2x - \pi \cos(\pi x)$

universelle Funktionen (ufunc) numpy Funktionen

[20]:
```
f_ufunc = lambdify(x, f_symfunk(x))
f_ufunc
```

[20]:
```
<function _lambdifygenerated(x)>
```

[21]:
```
f_ufunc(2)
```

[21]: 4.0

[22]:
```
z = symbols('z')
f_ufunc2 = lambdify(z, f_symfunk(z))
```

[23]:
```
f_ufunc2(2)
```

[23]: 4.0

[24]:
```
f_ufunc(np.array([1, .5, 3])), f_ufunc2(np.array([1, .5, 3]))
```

[24]:
```
(array([ 1.  , -0.75,  9.  ]), array([ 1.  , -0.75,  9.  ]))
```

1.2 Polynome I

[25]:
```
p = x**2 + 3
q = 2*x + 2
display(p)
display(q)
```

$x^2 + 3$

$2x + 2$

[26]:
```
d, r = div(p, q)
display(d)
display(r)
```

$\frac{x}{2} - \frac{1}{2}$

4

[27]:
```
s = x + y**2
t = 1
degree(s, x), degree(s, y) , degree(t, x)
```

[27]: $(1,\ 2,\ 0)$

1.3 Polynome II

[28]:
```
q = poly(2*x + 2)
q
```

[28]: $\text{Poly}\left(2x + 2, x, domain = \mathbb{Z}\right)$

[29]:
```
p = poly(p, x, domain=QQ)
p
```

[29]: $\text{Poly}\left(x^2 + 3, x, domain = \mathbb{Q}\right)$

[30]:
```
p = Poly([1, 0, 3], x) # Polynom aus der Liste der Koeffizienten
p
```

[30]: $\text{Poly}\left(x^2 + 3, x, domain = \mathbb{Z}\right)$

[31]:
```
s.as_poly()
```

[31]: $\text{Poly}\left(x + y^2, x, y, domain = \mathbb{Z}\right)$

[32]:
```
degree(s.as_poly(), y) # per Default wird der Grad bzgl. x ausgegeben
```

[32]: 2

1.3.1 Polynomdivision

[33]:
```
div(p, q)
```

[33]: $\left(\text{Poly}\left(\frac{1}{2}x - \frac{1}{2}, x, domain = \mathbb{Q}\right),\ \text{Poly}\left(4, x, domain = \mathbb{Q}\right)\right)$

[34]:
```
quo(p, q)
```

[34]: $\text{Poly}\left(\frac{1}{2}x - \frac{1}{2}, x, domain = \mathbb{Q}\right)$

[35]:
```
rem(p, q)
```

[35]: $\text{Poly}\left(4, x, domain = \mathbb{Z}\right)$

[36]:
```
p = poly(x**2+3, domain=ZZ)
p
```

[36]: $\text{Poly}\left(x^2 + 3, x, domain = \mathbb{Z}\right)$

[37]:
```
q = poly(2*x+2, x, domain = ZZ)
q
```

[37]: $\text{Poly}\left(2x + 2, x, domain = \mathbb{Z}\right)$

[38]:
```
div(p, q, domain = ZZ)
```

[38]: $\left(\text{Poly}\left(0, x, domain = \mathbb{Z}\right),\ \text{Poly}\left(x^2 + 3, x, domain = \mathbb{Z}\right)\right)$

1.3.2 Polynomgrad

[39]:
```
p = poly(2*x**3 + 3*x*y)
degree(p)
```

[39]: 3

[40]:
```
degree(p, x)
```

[40]: 3

[41]:
```
degree(p, y)
```

[41]: 1

[42]:
```
r = poly(0, x, domain=QQ)
r
```

[42]: $\text{Poly}\left(0, x, domain = \mathbb{Q}\right)$

[43]:
```
degree(r) # Achtung
```

[43]:
$$-\infty$$

1.3.3 Koeffizienten von Polynomen

[44]:
```
q = 3*x**2 + x - 1
```

[45]:
```
q.coeff(x, 2)
```

[45]:
$$3$$

[46]:
```
p = prod([x-j for j in range(5)])
p
```

[46]:
$$x\left(x-4\right)\left(x-3\right)\left(x-2\right)\left(x-1\right)$$

[47]:
```
p.expand().coeff(x, 4)
```

[47]:
$$-10$$

[48]:
```
n = 5
[p.expand().coeff(x, j) for j in range(n)]
```

[48]:
$$[0,\ 24,\ -50,\ 35,\ -10]$$

[49]:
```
p = Poly(p)
p
```

[49]:
$$\text{Poly}\left(x^5 - 10x^4 + 35x^3 - 50x^2 + 24x, x, domain = \mathbb{Z}\right)$$

[50]:
```
p.coeff(x, 4)
```

```
---------------------------------------------------------------------------
NotImplementedError                       Traceback (most recent call last)
/tmp/ipykernel_925273/629355135.py in <module>
----> 1 p.coeff(x, 4)

/local/home/schaedle/miniconda3/lib/python3.9/site-packages/sympy/polys/
 ↪polytools.py in coeff(f, x, n, right)
   2116         # at this time the ``right`` keyword would be ignored because␣
 ↪Poly
   2117         # doesn't work with non-commutatives.
-> 2118         raise NotImplementedError(
   2119             'Either convert to Expr with `as_expr` method '
   2120             'to use Expr\'s coeff method or else use the '

NotImplementedError: Either convert to Expr with `as_expr` method to use Expr's␣
 ↪coeff method or else use the `coeff_monomial` method of Polys.
```

[51]:
```
p.as_expr().coeff(x, 4)
```

[51]:
$$-10$$

[52]:
```
 p.coeff_monomial(x**4)
```

[52]:
$$-10$$

[53]:
```
p.all_coeffs()
```

[53]:
$$[1,\ -10,\ 35,\ -50,\ 24,\ 0]$$

[54]:
```
p.coeffs() # das ist nicht so hilfreich, da die 0 Koeffizienten fehlen
```

[54]:
$$[1,\ -10,\ 35,\ -50,\ 24]$$

1.3.4 Wurzeln von Polynomen / Nullstellen

[55]:
```
p = x**4 - x**2
roots(p)
```

[55]:
$$\{-1 : 1,\ 0 : 2,\ 1 : 1\}$$

[56]:
```
a, b, c = symbols('a b c')
roots(a*x**2 + b*x + c, x)
```

[56]:
$$\left\{-\frac{b}{2a} - \frac{\sqrt{-4ac+b^2}}{2a} : 1,\ -\frac{b}{2a} + \frac{\sqrt{-4ac+b^2}}{2a} : 1\right\}$$

[59]:
```
p = Poly(a*x**3+b*x**2+c*x+d, x)
roots(p, x)
```

[59]:
$$\left\{-\frac{-\frac{3(2c+1)}{2a} + \frac{b^2}{a^2}}{3\sqrt[3]{\frac{\sqrt{-4\left(-\frac{3(2c+1)}{2a}+\frac{b^2}{a^2}\right)^3+\left(-\frac{27}{2a}-\frac{9b(2c+1)}{2a^2}+\frac{2b^3}{a^3}\right)^2}}{2} - \frac{27}{4a} - \frac{9b(2c+1)}{4a^2} + \frac{b^3}{a^3}}} - \frac{\sqrt[3]{\frac{\sqrt{-4\left(-\frac{3(2c+1)}{2a}+\frac{b^2}{a^2}\right)^3+\left(-\frac{27}{2a}-\frac{9b(2c+1)}{2a^2}+\frac{2b^3}{a^3}\right)^2}}{2} - \right.}{3}$$

[62]:
```
p = Poly([1, -5, 4, 4 ,3, 9], x)
p
```

[62]:
$$\mathrm{Poly}\left(x^5 - 5x^4 + 4x^3 + 4x^2 + 3x + 9, x, domain = \mathbb{Z}\right)$$

[64]:
```
roots(p, x) # Glück gehabt
```

[64]:
$$\{-1 : 1,\ 3 : 2,\ -i : 1,\ i : 1\}$$

[65]:
```
real_roots(p, x)
```

[65]:

$[-1,\ 3,\ 3]$

[66]:
```
p = Poly(x**5-20*x +7)
roots(p) # Pech
```

[66]: $\{\}$

[107]:
```
for r in nroots(p):
    display(r)  # aber numerisch geht es
```

-2.19444446392152

0.350263599776741

2.01630911327803

$-0.0860641245666237 - 2.12350978358086i$

$-0.0860641245666237 + 2.12350978358086i$

1.4 Lösen von Gleichungen (solveset)

[67]:
```
gl = Eq((x-1)**2, 4-x)
gl
```

[67]: $(x-1)^2 = 4 - x$

[68]:
```
gl.lhs
```

[68]: $(x-1)^2$

[69]:
```
gl.rhs
```

[69]: $4 - x$

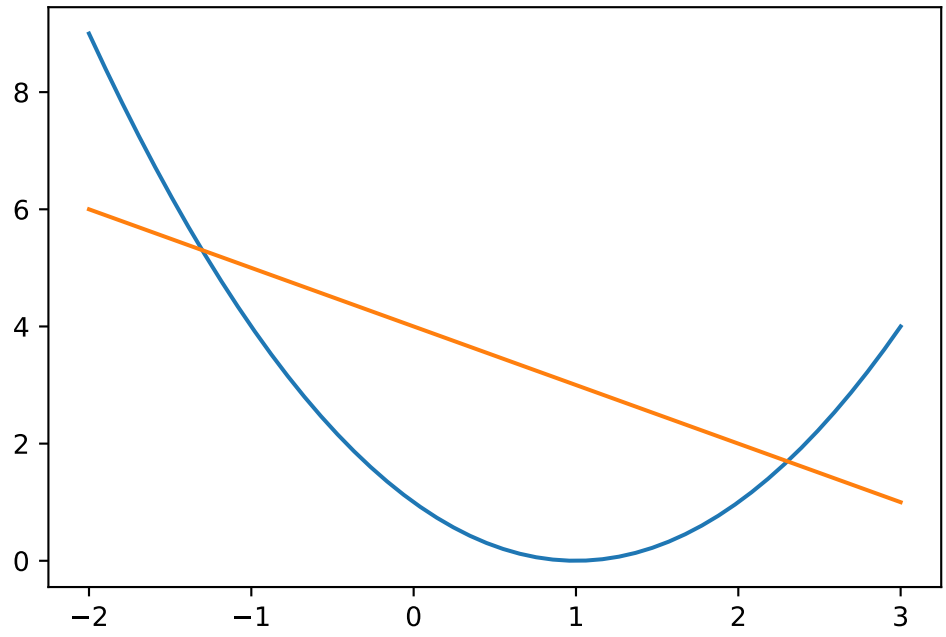
[70]:
```
Lsgn = solveset(gl, x)
type(Lsgn)
```

[70]: `sympy.sets.sets.FiniteSet`

[74]:
```
Lsgn
```

[74]: $\left\{\frac{1}{2} - \frac{\sqrt{13}}{2}, \frac{1}{2} + \frac{\sqrt{13}}{2}\right\}$

[71]:
```
fig = plt.figure()
ax = fig.gca()
xn = np.linspace(-2, 3)
ax.plot(xn,lambdify(x, gl.lhs)(xn))
ax.plot(xn,lambdify(x, gl.rhs)(xn))
plt.show()
```



[72]:
```
# Test durch einsetzen
for lsg in Lsgn:
    display(gl.subs(x, lsg))
```

True

True

[73]:
```
solveset((x-1)**2 - 4+x, x)  # loest (x-1)**2-4+x = 0
```

[73]: $\left\{\frac{1}{2} - \frac{\sqrt{13}}{2}, \frac{1}{2} + \frac{\sqrt{13}}{2}\right\}$

[75]:
```
glt = Eq(sin(x), cos(x))
glt
```

[75]: $\sin(x) = \cos(x)$

[76]:
```
Lsg = solveset(glt)
Lsg
```

[76]: $\left\{2n\pi + \frac{5\pi}{4} \,\middle|\, n \in \mathbb{Z}\right\} \cup \left\{2n\pi + \frac{\pi}{4} \,\middle|\, n \in \mathbb{Z}\right\}$

[77]:
```
for i, lsg in zip(range(7), Lsg):
    display(lsg)
```

$$\frac{5\pi}{4}$$

$$\frac{\pi}{4}$$

$$\frac{13\pi}{4}$$

$$\frac{9\pi}{4}$$

$$-\frac{3\pi}{4}$$

$$-\frac{7\pi}{4}$$

$$\frac{21\pi}{4}$$

[78]:
```
print(Lsg)
```

```
Union(ImageSet(Lambda(_n, 2*_n*pi + 5*pi/4), Integers), ImageSet(Lambda(_n,
2*_n*pi + pi/4), Integers))
```

[79]:
```
glt = Eq(sin(2*x), cos(x))
glt
```

[79]:
$$\sin(2x) = \cos(x)$$

[80]:
```
Lsg = solveset(glt, x)
Lsg
```

[80]:
$$\left\{2n\pi + \frac{\pi}{2} \,\middle|\, n \in \mathbb{Z}\right\} \cup \left\{2n\pi + \frac{3\pi}{2} \,\middle|\, n \in \mathbb{Z}\right\} \cup \left\{2n\pi + \frac{5\pi}{6} \,\middle|\, n \in \mathbb{Z}\right\} \cup \left\{2n\pi + \frac{\pi}{6} \,\middle|\, n \in \mathbb{Z}\right\}$$

[81]:
```
Lsg = solveset(glt, x, Interval(-5,10)) #Lösung in einem Intervall
Lsg
```

[81]:
$$\left\{-\frac{3\pi}{2}, -\frac{7\pi}{6}, -\frac{\pi}{2}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{13\pi}{6}, \frac{5\pi}{2}, \frac{17\pi}{6}\right\}$$

[82]:
```
glt = Eq(8*sin(x), x+1)
glt
```

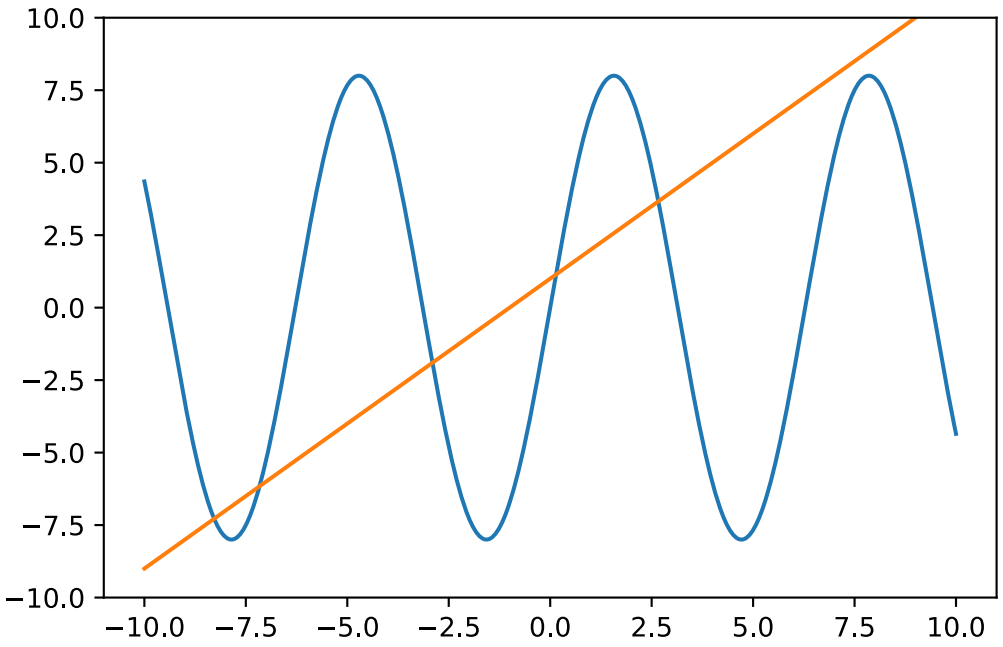
[82]:
$$8\sin(x) = x + 1$$

[83]:
```
Lsgt = solveset(glt)
Lsgt
```

[83]:
$$\{x \mid x \in \mathbb{C} \wedge -x + 8\sin(x) - 1 = 0\}$$

[84]:
```
gl = glt.lhs
gr = glt.rhs
xn = np.linspace(-10, 10, 500)
gln = lambdify(x, gl)
grn = lambdify(x, gr)
fig = plt.figure()
plt.plot(xn, gln(xn), xn, grn(xn))
plt.ylim((-10, 10))
```

[84]: $(-10.0,\ 10.0)$



[85]:
```
solveset(exp(x), x) # exp(x) == 0
```

[85]: $\emptyset$

[86]:
```
solveset(Eq(exp(x**2), 1)) # exp(x**2) = 1
```

[86]: $\left\{x \,\middle|\, x \in \mathbb{C} \wedge e^{x^2} - 1 = 0\right\}$

[87]:
```
solveset(exp(x)+x, x)
```

[87]: $\{x \mid x \in \mathbb{C} \wedge x + e^{x} = 0\}$

[88]:
```
solveset(x**2+1, x, domain=Reals)
```

[88]: $\emptyset$

[89]:
```
solveset(log(x+1)+log(x-2), x) # log(x+1)+log(x-2) = 0
```

[89]:
$$\left\{\frac{1}{2} - \frac{\sqrt{13}}{2}, \frac{1}{2} + \frac{\sqrt{13}}{2}\right\}$$

[90]:
```
solveset(Eq(5**(x-2), 3**(2*x+1)), x) # 5**(x-2) = 3**(2*x+1)
```

[90]:
$$\left\{-\frac{\log(3) + 2\log(5)}{-\log(5) + 2\log(3)}\right\}$$