lektion8

January 29, 2018

Table of Contents

1 Lektion 8

1.1 Vereinfachungen (simplify)

In [1]:
```
from sympy import *
init_printing()
x,y,z,a,b,c = symbols('x y z a b c')
```

In [2]:
```
f = sin(x)**2 + cos(x)**2
f
```

Out[2]:

$$\sin^2(x) + \cos^2(x)$$

In [3]:
```
simplify(f)
```

Out[3]:

$$1$$

In [4]:
```
p= (x**3 + x**2 +x +1) / (x**2 +2* x +1)
p
```

Out[4]:

$$\frac{x^3 + x^2 + x + 1}{x^2 + 2x + 1}$$

```
In [5]: simplify(p)
```

Out[5]:

$$\frac{x^2+1}{x+1}$$

```
In [6]: g = exp((x-1)**2+log(c*exp(y**2)-exp(4*x))-(x+1)**2)
        g
```

Out[6]:

$$\left(ce^{y^2}-e^{4x}\right)e^{(x-1)^2-(x+1)^2}$$

```
In [7]: simplify(g)
```

Out[7]:

$$ce^{-4x+y^2}-1$$

```
In [8]: q = simplify( x**2+2*x+1 )
        q
```

Out[8]:

$$x^2+2x+1$$

1.1.1 faktorisieren (factor)

```
In [9]: h = factor(q)
        h
```

Out[9]:

$$(x+1)^2$$

1.1.2 ausmultiplizieren (expand)

```
In [10]: q = expand(h)
         q
```

Out[10]:

$$x^2+2x+1$$

```
In [11]: f = (x+1)*(x-2)-(x-1)*x+2
         f
```

Out[11]:

$$-x(x-1)+(x-2)(x+1)+2$$

```
In [12]: expand(f)
```

Out[12]:

$$0$$

```
In [13]: f = (x**2-y**2)/(x+y)**2
         f
```

Out[13]:

$$\frac{x^2 - y^2}{(x+y)^2}$$

```
In [14]: factor(f)
```

Out[14]:

$$\frac{x-y}{x+y}$$

```
In [15]: expand(f)
```

Out[15]:

$$\frac{x^2}{x^2 + 2xy + y^2} - \frac{y^2}{x^2 + 2xy + y^2}$$

1.1.3 "cancel" bringt rationale Ausdruecke in gekuerzte Standardform

```
In [16]: cancel(f)
```

Out[16]:

$$\frac{x-y}{x+y}$$

```
In [17]: g = (exp(-x)-exp(x))**3
         g
```

Out[17]:

$$\left(-e^x + e^{-x}\right)^3$$

```
In [18]: f = expand(g)
         f
```

Out[18]:

$$-e^{3x} + 3e^x - 3e^{-x} + e^{-3x}$$

```
In [19]: factor(f)
```

Out[19]:

$$-\left(e^{x}-1\right)^{3}\left(e^{x}+1\right)^{3}e^{-3x}$$

```
In [20]: f = integrate(x**2 * (exp(x)+exp(-x)),x)
         f
```

Out[20]:

$$\left(-x^{2}-2x-2\right)e^{-x}+\left(x^{2}-2x+2\right)e^{x}$$

```
In [21]: simplify(f)
```

Out[21]:

$$\left(-x^{2}-2x+\left(x^{2}-2x+2\right)e^{2x}-2\right)e^{-x}$$

```
In [22]: g = factor(f)
         g
```

Out[22]:

$$\left(x^{2}e^{2x}-x^{2}-2xe^{2x}-2x+2e^{2x}-2\right)e^{-x}$$

```
In [23]: h = expand(g)
         h
```

Out[23]:

$$x^{2}e^{x}-x^{2}e^{-x}-2xe^{x}-2xe^{-x}+2e^{x}-2e^{-x}$$

1.1.4 Zusammenfassen (collect)

```
In [24]: collect(h,x)
```

Out[24]:

$$x^{2}\left(e^{x}-e^{-x}\right)+x\left(-2e^{x}-2e^{-x}\right)+2e^{x}-2e^{-x}$$

```
In [25]: collect(h,exp(x))
```

Out[25]:

$$\left(-x^{2}-2x-2\right)e^{-x}+\left(x^{2}-2x+2\right)e^{x}$$

```
In [26]: collect(h,exp(x),exact=True)
```

Out[26]:

$$-x^{2}e^{-x}-2xe^{-x}+\left(x^{2}-2x+2\right)e^{x}-2e^{-x}$$

```
In [27]: collect(h,x**2,exact=True)
```

Out[27]:

$$x^2\left(e^x - e^{-x}\right) - 2xe^x - 2xe^{-x} + 2e^x - 2e^{-x}$$

```
In [28]: k = 1+x/(x -2/(x-4/(8-x))) # Kettenbruch
         k
```

Out[28]:

$$\frac{x}{x - \frac{2}{x - \frac{4}{-x+8}}} + 1$$

```
In [29]: cancel(k)
```

Out[29]:

$$\frac{2x^3 - 16x^2 + 6x + 16}{x^3 - 8x^2 + 2x + 16}$$

```
In [30]: simplify(k)
```

Out[30]:

$$\frac{x}{x - \frac{2}{x + \frac{4}{x-8}}} + 1$$

```
In [31]: factor(k) # das gleiche wie cancel
```

Out[31]:

$$\frac{2\left(x^3 - 8x^2 + 3x + 8\right)}{x^3 - 8x^2 + 2x + 16}$$

1.1.5 Partialbruchzerlegung

```
In [32]: h = apart(k) #Partialbruchzerlegung
         h
```

Out[32]:

$$\frac{2(x-8)}{x^3 - 8x^2 + 2x + 16} + 2$$

```
In [33]: together(h)
```

Out[33]:

$$\frac{2\left(x^3 - 8x^2 + 3x + 8\right)}{x^3 - 8x^2 + 2x + 16}$$

In [34]:
```
f = log(y/x)-log(y)+log(x)
f
```

Out[34]:

$$\log{\left (x \right)} - \log{\left (y \right)} + \log{\left (\frac{y}{x} \right)}$$

1.2 Vereinfachung unter Annahmen (assumptions)

In [35]:
```
simplify(f)
```

Out[35]:

$$\log{\left (x \right)} - \log{\left (y \right)} + \log{\left (\frac{y}{x} \right)}$$

In [36]:
```
x,y = symbols('x y',positive=True)
f = log(y/x)-log(y)+log(x)
simplify(f)
```

Out[36]:

$$0$$

In [37]:
```
x,y = symbols('x y')
x.assumptions0, y.assumptions0
```

Out[37]:
```
({'commutative': True}, {'commutative': True})
```

In [38]:
```
f = sin(x)**4 - 2*sin(x)**2*cos(x)**2 + cos(x)**4
f
```

Out[38]:

$$\sin^{4}{\left (x \right)} - 2 \sin^{2}{\left (x \right)} \cos^{2}{\left (x \right)} + \cos^{4}{\left (x \right)}$$

In [39]:
```
simplify(f)
```

Out[39]:

$$\frac{1}{2} \cos{\left (4 x \right)} + \frac{1}{2}$$

1.3 trigsimp und powsimp

In [40]:
```
trigsimp(f)
```

Out[40]:

$$\frac{1}{2} \cos{\left (4 x \right)} + \frac{1}{2}$$

In [41]:
```
simplify(sinh(x)**2+cosh(x)**2), trigsimp(sinh(x)**2+cosh(x)**2)
```

Out[41]:

$$\left(\cosh\left(2x\right),\quad \cosh\left(2x\right)\right)$$

In [42]:
```
simplify(cos(x+y))
```

Out[42]:

$$\cos\left(x+y\right)$$

In [43]:
```
trigsimp(cos(x+y))
```

Out[43]:

$$\cos\left(x+y\right)$$

In [44]:
```
expand(cos(x+y))
```

Out[44]:

$$\cos\left(x+y\right)$$

In [45]:
```
expand_trig(cos(x+y))
```

Out[45]:

$$-\sin\left(x\right)\sin\left(y\right)+\cos\left(x\right)\cos\left(y\right)$$

In [46]:
```
expand_trig(sinh(x+y))
```

Out[46]:

$$\sinh\left(x\right)\cosh\left(y\right)+\sinh\left(y\right)\cosh\left(x\right)$$

In [47]:
```
f = expand_trig(tan(4*x))
f
```

Out[47]:

$$\frac{-4\tan^{3}\left(x\right)+4\tan\left(x\right)}{\tan^{4}\left(x\right)-6\tan^{2}\left(x\right)+1}$$

In [48]:
```
trigsimp(f)
```

Out[48]:

$$\tan\left(4x\right)$$

In [49]:
```
simplify(x**a*x)
```

Out[49]:

$$x^{a+1}$$

```
In [50]: powsimp(x**a*x**b)
```

Out[50]:

$$x^{a+b}$$

```
In [51]: trigsimp(x**a*x*sin(x)/cos(x))
```

Out[51]:

$$xx^{a}\tan(x)$$

```
In [52]: simplify(x**a*x*sin(x)/cos(x))
```

Out[52]:

$$x^{a+1}\tan(x)$$

```
In [53]: powsimp(x**a*x*sin(x)/cos(x))
```

Out[53]:

$$\frac{x^{a+1}\sin(x)}{\cos(x)}$$

```
In [54]: powsimp(x**a*y**a) # Uebungen
```

Out[54]:

$$x^{a}y^{a}$$

```
In [55]: powsimp((x**a)**b)
```

Out[55]:

$$\left(x^{a}\right)^{b}$$

1.4 Umformungen (rewrite)

```
In [56]: sin(2*x).rewrite(cot)
```

Out[56]:

$$\frac{2\cot(x)}{\cot^{2}(x)+1}$$

```
In [57]: sin(2*x).rewrite(cos)
```

Out[57]:

$$\cos\left(2x - \frac{\pi}{2}\right)$$

```
In [58]: sin(2*x).rewrite(exp)
```

Out[58]:

$$-\frac{i}{2}\left(e^{2ix} - e^{-2ix}\right)$$

```
In [59]: cot(x+1).rewrite(tan)
```

Out[59]:

$$\frac{1}{\tan\left(x+1\right)}$$

```
In [60]: tan(x).rewrite(sin)
```

Out[60]:

$$\frac{2\sin^{2}\left(x\right)}{\sin\left(2x\right)}$$

```
In [61]: besselj(Rational(1,2),x).rewrite(sin)
```

Out[61]:

$$J_{\frac{1}{2}}\left(x\right)$$

```
In [62]: plot(besselj(Rational(1,2),x)-sin(x)*sqrt(2/pi/x),(x,0,2));
```

```
<matplotlib.figure.Figure at 0x7f9a1a8656a0>
```

gamma(4)

```
In [63]: gamma(5)
```

Out[63]:

$$24$$

```
In [64]: f = gamma(x)*gamma(x+Rational(1,2)) -2**(1-2*x)*sqrt(pi)*gamma(2*x)
         f
```

Out[64]:

$$-2^{-2x+1}\sqrt{\pi}\Gamma(2x) + \Gamma(x)\Gamma\left(x + \frac{1}{2}\right)$$

```
In [65]: simplify(f)
```

Out[65]:

$$0$$

1.5 Anwendungsbeispiel Kettenbruch (nicht klausurrelevant)

In [66]:
```
n=9
def PadeSqrtB(n):
    qb = [1]
    pb = [0]
    for ii in range(1,n+1):
        qb.append( 1 + (ii+1) % 2 )
        pb.append(z/2)
    return qb,pb
```

In [67]:
```
qb, pb = PadeSqrtB(n)
display(qb)
display(pb)
```

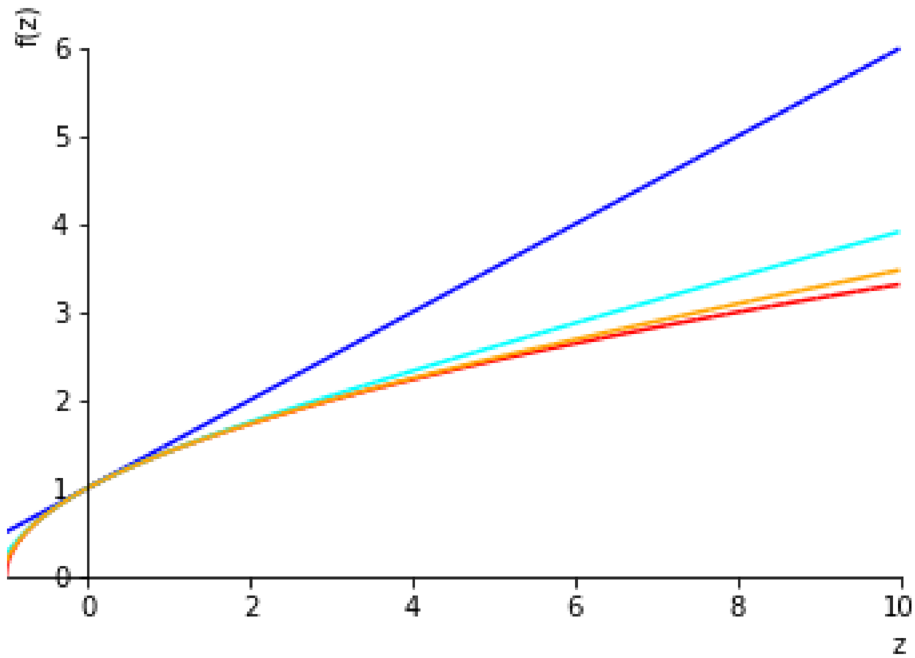
$$[1, \ 1, \ 2, \ 1, \ 2, \ 1, \ 2, \ 1, \ 2, \ 1]$$

$$\left[0, \ \frac{z}{2}, \ \frac{z}{2}, \ \frac{z}{2}, \ \frac{z}{2}, \ \frac{z}{2}, \ \frac{z}{2}, \ \frac{z}{2}, \ \frac{z}{2}, \ \frac{z}{2}\right]$$

In [68]:
```
Ab = {-1 : 1,
    0 : qb[0]}
Bb = {-1: 0,
    0 : 1}
Rb = {}

for i in range(1,n+1):
  Ab[i] = simplify(qb[i]*Ab[i-1]+pb[i]*Ab[i-2]);
  Bb[i] = simplify(qb[i]*Bb[i-1]+pb[i]*Bb[i-2]);
  Rb[i] = cancel(Ab[i]/Bb[i]);
```

In [69]:
```
fg = plot(sqrt(1+z),Rb[1],Rb[3],Rb[5],(z,-1,10),show=False)
fg[0].line_color='red'
fg[1].line_color='blue'
fg[2].line_color='cyan'
fg[3].line_color='orange'
fg.show()
```




```
In [70]: h = symbols('h:{0:d}'.format(int(n/2+2)))
         k = symbols('k:{0:d}'.format(int(n/2+2)))

In [71]: qd = {0 : 0}
         pd = {}
         nh = int((n+1)/2)
         for i in range(1, nh+1):
           qd.update({ 2*i-1: k[i]*(1+z)})
           qd.update({ 2*i : h[i]})

         for i in range(1,n+2):
           pd.update({i :1})

         Ad= {-1 : 1,
             0 : qd[0],
             1 : qd[1]*qd[0]+pd[1]
           }

         Bd = {-1 : 0,
             0 : 1,
             1 : qd[1]
           }

         Rd = {}
         for i in range(2, n+2):
```

```
        Ad[i] = factor(qd[i]*Ad[i-1]+pd[i]*Ad[i-2])
        Bd[i] = factor(qd[i]*Bd[i-1]+pd[i]*Bd[i-2])
        Rd[i] = cancel(Bd[i]/Ad[i])
```

```
In [72]: m = 5; # ungerade Zahl kleiner als n
         eq1 = collect(denom(Rd[m+1])-denom(Rb[m]),z,factor).as_poly(z).all_coeffs()
         eq2 = collect(numer(Rd[m+1])-numer(Rb[m]),z,factor).as_poly(z).all_coeffs()
         eq1.extend(eq2)
         solve(eq1)
```

Out[72]:

$$\left[\left\{h_1 : \frac{18}{35},\quad h_2 : \frac{64}{45},\quad h_3 : \frac{256}{63},\quad k_1 : \frac{1}{6},\quad k_2 : \frac{175}{192},\quad k_3 : \frac{567}{256}\right\}\right]$$