lektion6

January 29, 2018

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1 Lektion 6

In [1]:
```
from sympy import *
init_printing()
import matplotlib.pyplot as plt
import numpy as np
%matplotlib notebook
#%matplotlib inline
x,y,z,a,b,c,d = symbols('x y z a b c d')
```

1.1 Polynome

In [2]:
```
p = 2*x**2 + 3
q = x+1
d,r = div(p,q)
d,r
```

Out[2]:

$$(2x - 2, \quad 5)$$

In [3]:
```
q = x+y**2
p = 1
degree(q,x), degree(q,y), degree(q) #, degree(p)
```

Out[3]:

$$(1, \quad 2, \quad 1)$$

besser

In [4]:
```
p = poly(2*x+y**2,domain=QQ)
p
```

Out[4]:

$$\text{Poly}\left(2x + y^2, x, y, domain = \mathbb{Q}\right)$$

```
In [5]: q = poly(0,x,domain=QQ)
        #q = poly(1,x,domain=QQ)
        q
```

Out[5]:

$$\text{Poly}\left(0, x, domain = \mathbb{Q}\right)$$

```
In [6]: degree(q) # Achtung
```

Out[6]:

$$-\infty$$

1.2 Loesen von Gleichungen (solve)

```
In [7]: gl = Eq((x-1)**2,4-x)
        gl
```

Out[7]:

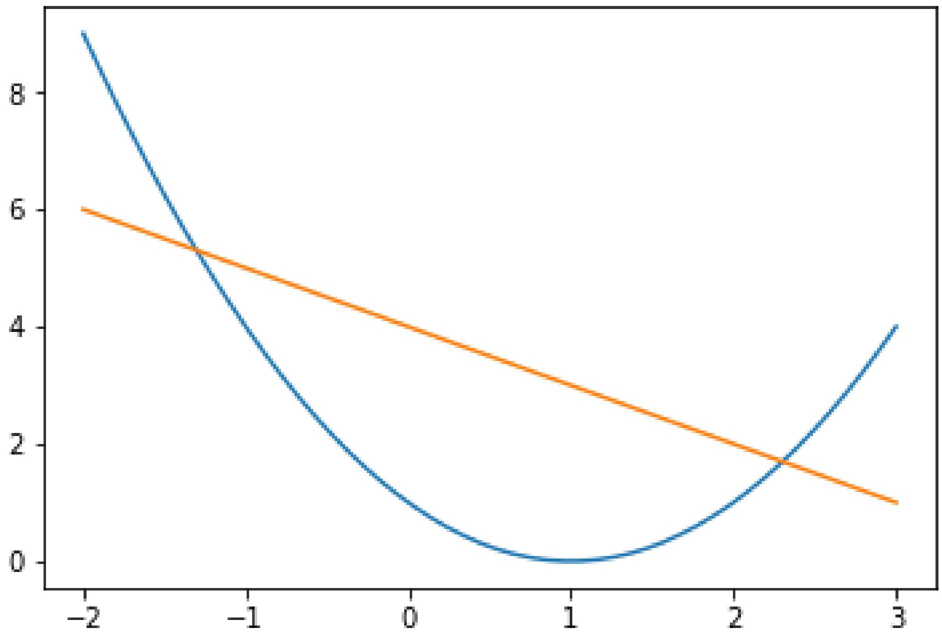
$$(x-1)^2 = -x + 4$$

```
In [8]: lsg = solve(gl,x)
        lsg
```

Out[8]:

$$\left[\frac{1}{2} + \frac{\sqrt{13}}{2}, \quad -\frac{\sqrt{13}}{2} + \frac{1}{2}\right]$$

```
In [9]: fig = plt.figure()
        ax = fig.gca()
        xn = np.linspace(-2,3)
        ax.plot(xn,lambdify(x,gl.lhs)(xn))
        ax.plot(xn,lambdify(x,gl.rhs)(xn))
        plt.show()
```




```
In [10]: gl.subs(x,lsg[0])
```

Out[10]:

True

```
In [11]: lsg = solve({gl})
         lsg
```

Out[11]:

$$\left[\left\{ x : \frac{1}{2} + \frac{\sqrt{13}}{2} \right\}, \quad \left\{ x : -\frac{\sqrt{13}}{2} + \frac{1}{2} \right\} \right]$$

```
In [12]: gl.subs(x,lsg[0][x])
```

Out[12]:

True

```
In [13]: gl.subs(lsg[0])
```

Out[13]:

True

```
In [14]: solve(a*x**2+b*x+c,x)
```

Out[14]:

$$\left[\frac{1}{2a}\left(-b+\sqrt{-4ac+b^{2}}\right),\quad -\frac{1}{2a}\left(b+\sqrt{-4ac+b^{2}}\right)\right]$$

In [15]:
```
sol = solve(a*x**3+b*x**2+c*x+d,x) #Cardano
sol
```

Out[15]:

$$\left[-\frac{-\frac{3}{a}(c+2)+\frac{b^{2}}{a^{2}}}{3\sqrt[3]{\frac{1}{2}\sqrt{-4\left(-\frac{3}{a}(c+2)+\frac{b^{2}}{a^{2}}\right)^{3}+\left(-\frac{54}{a}-\frac{9b}{a^{2}}(c+2)+\frac{2b^{3}}{a^{3}}\right)^{2}}-\frac{27}{a}-\frac{9b}{2a^{2}}(c+2)+\frac{b^{3}}{a^{3}}}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\sqrt{-4\left(-\frac{3}{a}(c+2)\right.}}\right.$$

In [16]:
```
p = Eq(x**3-5*x**2+3,0)
Lsg = solve(p)
Lsg
```

Out[16]:

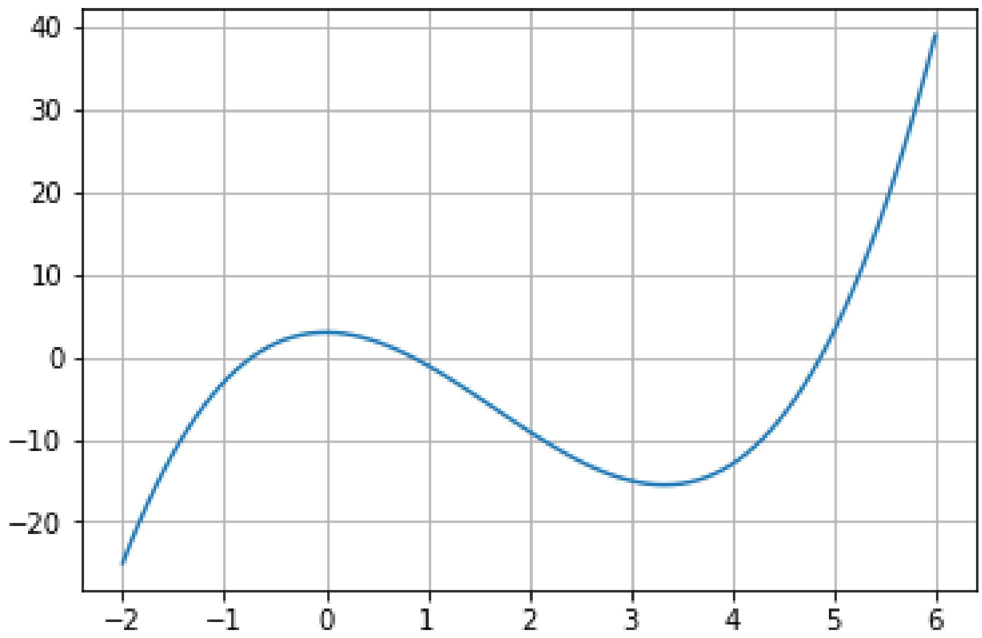
$$\left[\frac{5}{3}+\left(-\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)\sqrt[3]{\frac{169}{54}+\frac{\sqrt{419}i}{6}}+\frac{25}{9\left(-\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)\sqrt[3]{\frac{169}{54}+\frac{\sqrt{419}i}{6}}},\quad \frac{5}{3}+\frac{25}{9\left(-\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)\sqrt[3]{\frac{169}{54}+\frac{\sqrt{419}i}{6}}}+\left(-\frac{1}{2}+\right.\right.$$

In [17]:
```
[l.n() for l in Lsg]
```

Out[17]:

$$\left[0.850256587242986-1.0\cdot 10^{-22}i,\quad -0.723956489491132+5.0\cdot 10^{-23}i,\quad 4.87369990224815-3.0\cdot 10^{-21}i\right]$$

In [18]:
```
fig = plt.figure()
ax = fig.gca()
xn = np.linspace(-2,6,100)
ax.plot(xn,lambdify(x,p.lhs)(xn))
ax.grid()
```



1.3 Wurzeln von Polynomen

```
In [19]: wurzeln = roots(p,trig=True)
         wurzeln
```

Out[19]:

$$\left\{ \frac{5}{3} + \frac{10}{3}\cos\left(\frac{1}{3}\operatorname{acos}\left(\frac{169}{250}\right)\right) : 1, \quad -\frac{10}{3}\sin\left(-\frac{1}{3}\operatorname{acos}\left(\frac{169}{250}\right) + \frac{\pi}{6}\right) + \frac{5}{3} : 1, \quad -\frac{10}{3}\cos\left(-\frac{1}{3}\operatorname{acos}\left(\frac{169}{250}\right) + \frac{\pi}{3}\right)$$

```
In [20]: [l.n() for l in Lsg], [w.n() for w in wurzeln ]
```

Out[20]:

$$\left(\left[0.850256587242986 - 1.0 \cdot 10^{-22} i, \quad -0.723956489491132 + 5.0 \cdot 10^{-23} i, \quad 4.87369990224815 - 3.0 \cdot 10^{-21} i\right], \quad [4$$

```
In [21]: [p.subs(x,l).simplify() for l in Lsg]
```

Out[21]:

$$[\text{True}, \quad \text{True}, \quad \text{True}]$$

```
In [22]: [p.subs(x,w).expand(trig=True).trigsimp() for w in wurzeln]
```

Out[22]:

$$[\mathrm{True}, \quad \mathrm{True}, \quad \mathrm{True}]$$

In [23]:
```python
q = Eq(1*x**4-2*x**3-3*x**2+5*x+1,0)
sol = solve(q,x)
print([l.n() for l in sol])
[w.simplify() for w in roots(q)]
```

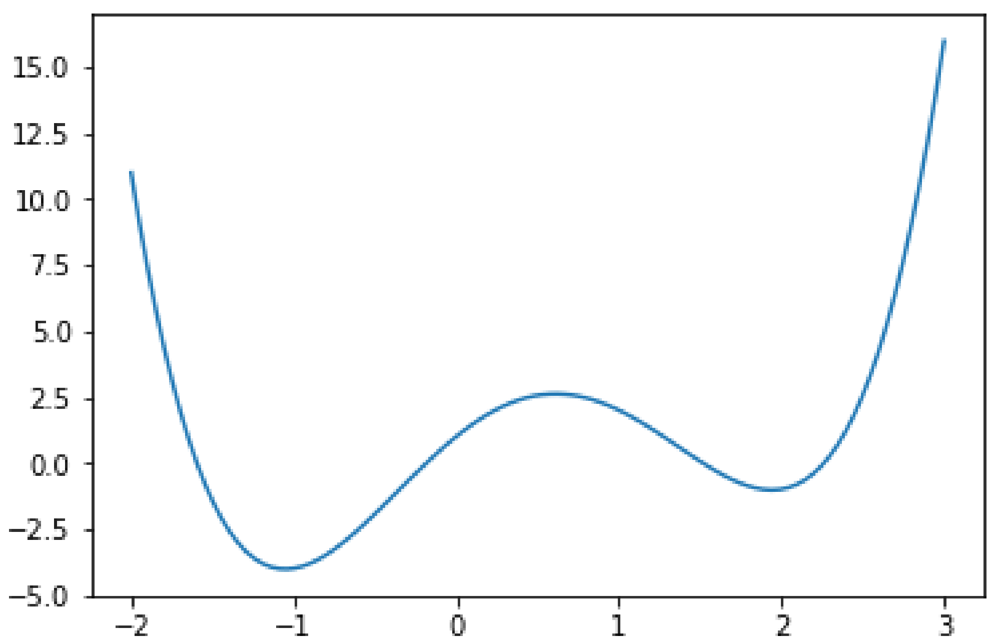
```
[-1.59615467600863 + 5.48401685356915e-31*I, 1.51572158929134 + 2.67884293265673e-30*I, 2.263077
```

Out[23]:

$$\left[\frac{1}{2} - \frac{1}{12}\sqrt{108 + \frac{204\sqrt[3]{18}}{\sqrt[3]{225 + \sqrt{8331}i}} + 6\sqrt[3]{12}\sqrt[3]{225 + \sqrt{8331}i}} + \frac{1}{12}\sqrt{216 - 6\sqrt[3]{12}\sqrt[3]{225 + \sqrt{8331}i} + \frac{\quad}{\sqrt{18 + \frac{34\sqrt[3]{18}}{\sqrt[3]{225+\sqrt{83}}}}}}\right.$$

In [24]:
```python
fig = plt.figure()
ax = fig.gca()
xn = np.linspace(-2,3,100)
ax.plot(xn,lambdify(x,q.lhs)(xn))
```

Out[24]: [<matplotlib.lines.Line2D at 0x7fcf1e029358>]



```
In [25]: lsg = solve(x**5-x-11) # -> Algebra
         lsg
```

Out[25]:

$$\left[\operatorname{CRootOf}\left(x^5 - x - 11, 0\right), \quad \operatorname{CRootOf}\left(x^5 - x - 11, 1\right), \quad \operatorname{CRootOf}\left(x^5 - x - 11, 2\right), \quad \operatorname{CRootOf}\left(x^5 - x - 11, 3\right),\right.$$

```
In [26]: [l.n() for l in lsg]
```

Out[26]:

$$[1.66148698080144, \quad -1.2926835755914 - 0.903032173152019i, \quad -1.2926835755914 + 0.903032173152019i, \quad 0.46$$

```
In [27]: n= int(3)
         a=Rational(-1,2)
         b=3
         f = 1/(2**n*factorial(n)) *(1-x)**(-a) *(1+x)**(-b) * diff((1-x)**a * (1+x)**b * (1-x**
         f
         #f.simplify()
         #plot(f,(x,-1/2,3/2))
```

Out[27]:

$$-\frac{1}{16\left(x+1\right)^{3}}\left(16x^{3}\left(x+1\right)^{3}+72x^{2}\left(x+1\right)^{2}\left(x^{2}-1\right)+\frac{12x^{2}\left(x+1\right)^{3}\left(x^{2}-1\right)}{-x+1}+24x\left(x+1\right)^{3}\left(x^{2}-1\right)+36x\left(x+\right.\right.$$

```
In [28]: sols = solve(f.simplify())
         sols
```

Out[28]:

$$\left[\frac{7}{17}-\frac{96}{289\left(-\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)\sqrt[3]{\frac{24192}{63869}+\frac{3456i}{3757}}}-\frac{1}{3}\left(-\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)\sqrt[3]{\frac{24192}{63869}+\frac{3456i}{3757}}, \quad \frac{7}{17}-\frac{1}{3}\left(-\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)\sqrt[3]{\frac{24192}{63869}+\frac{3}{3}}\right.$$

```
In [29]: [im(sol).simplify() for sol in sols]
```

Out[29]:

$$[0, \quad 0, \quad 0]$$

```
In [30]: [re(sol).simplify() for sol in sols]
```

Out[30]:

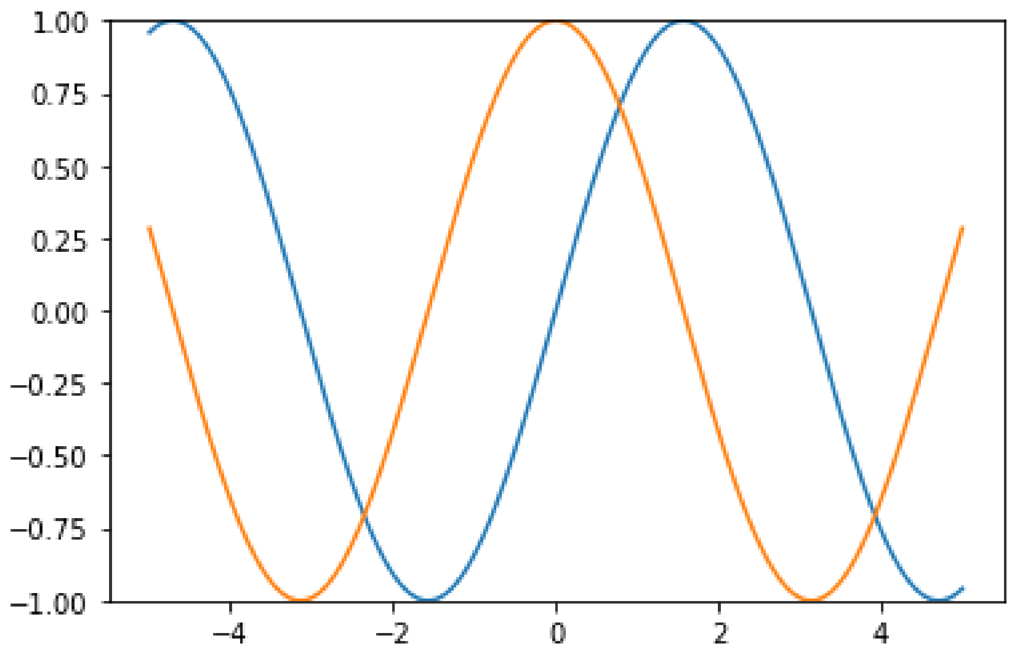
$$\left[\frac{8\sqrt{2}}{17}\sin\left(\frac{1}{6}\left(-2\operatorname{atan}\left(\frac{17}{7}\right)+\pi\right)\right)+\frac{7}{17}, \quad \frac{7}{17}+\frac{8\sqrt{2}}{17}\sin\left(\frac{1}{6}\left(2\operatorname{atan}\left(\frac{17}{7}\right)+\pi\right)\right), \quad -\frac{8\sqrt{2}}{17}\cos\left(\frac{1}{3}\operatorname{atan}\left(\frac{17}{7}\right.\right.\right.$$

In [31]:
```python
gl = Eq(sin(x),cos(x))
gl
```

Out[31]:

$$\sin(x) = \cos(x)$$

In [32]:
```python
def glp(lb,ub,gl,nn=500):
    fig = plt.figure()
    ax = fig.gca()
    xn = np.linspace(lb,ub,nn)
    ax.plot(xn,lambdify(x,gl.lhs)(xn))
    ax.plot(xn,lambdify(x,gl.rhs)(xn))
    #plt.show()
    return fig, ax
fig, ax = glp(-5,5,gl)
ax.set_ylim((-1,1));
```



In [33]:
```python
solve(gl,x)
```

Out[33]:

$$\left[-\frac{3\pi}{4}, \quad \frac{\pi}{4} \right]$$

In [34]:
```python
solveset(gl,x)
```

Out[34]:

$$\left\{2n\pi + \frac{5\pi}{4} \mid n \in \mathbb{Z}\right\} \cup \left\{2n\pi + \frac{\pi}{4} \mid n \in \mathbb{Z}\right\}$$

```
In [35]: solveset(exp(x),x) # exp(x) == 0
```

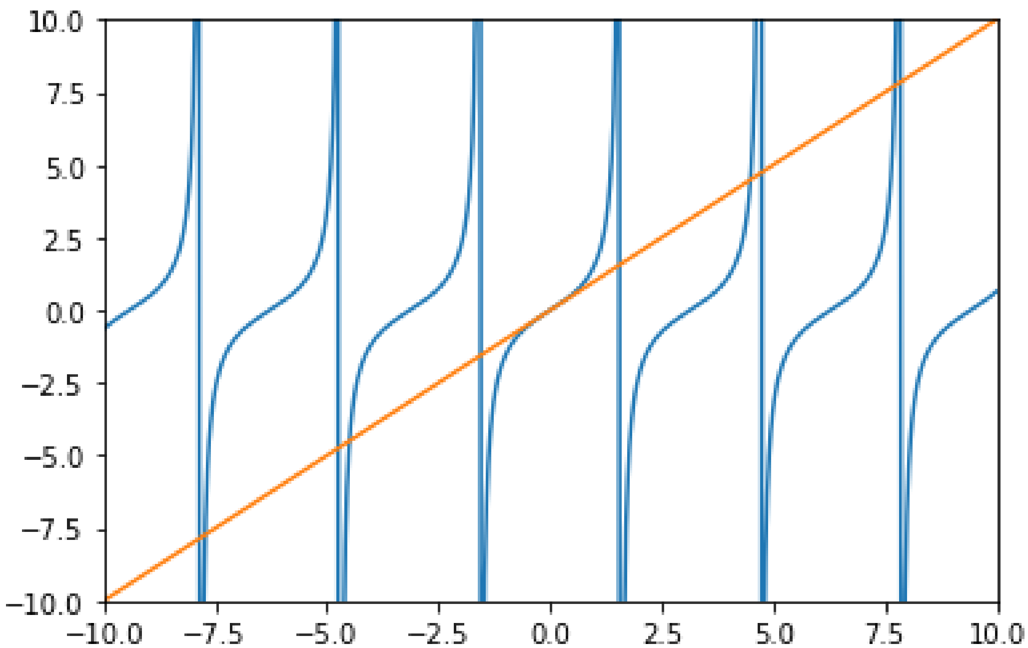
Out[35]:

$$\emptyset$$

```
In [36]: gl = Eq(tan(x),x)
         fig, ax = glp(-10,10,gl)
         ax.set_ylim([-10,10])
         ax.axis([-10,10,-10,10])
```

Out[36]:

$$[-10, \quad 10, \quad -10, \quad 10]$$




```
In [37]: solveset(gl)
```

Out[37]:

$$\{x \mid x \in \mathbb{C} \wedge -x + \tan(x) = 0\}$$

1.4 Numerische Loesung nichtlinearer Gleichungen

In [38]:
```
nsolve(gl,1)
```

Out[38]:

0.000348227174421857

In [39]:
```
nsolve(gl,(np.pi, 1.499*np.pi),solver='bisect')
```

Out[39]:

4.49340945790906