lektion14

January 29, 2018

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```
In [1]: from IPython.display import display
        import numpy as np
        import matplotlib.pyplot as plt
        from sympy import *
        init_printing()
        %matplotlib inline
        x,y,z,mu = symbols('x y z mu')
```

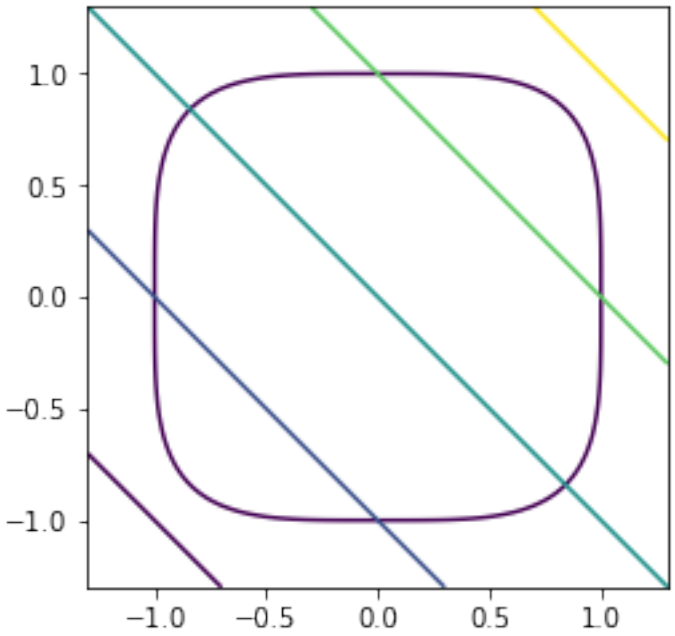
0.1 Extrema unter Nebenbedingungen

```
In [2]: g = x**4+y**4-1
        gn = lambdify((x,y),g)
        gn(1,2)
```

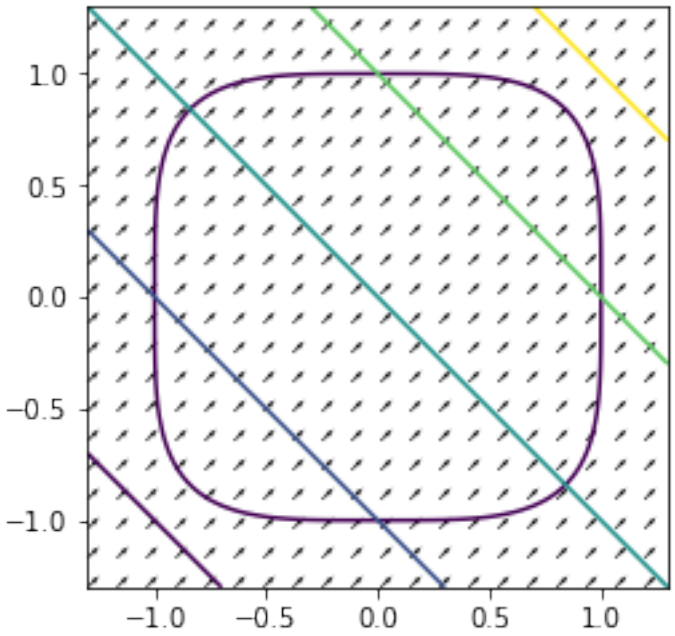
```
Out[2]:
```

16

```
In [3]: g = x**4+y**4-1
        f = x+y
        gn = lambdify((x,y),g)
        fn = lambdify((x,y),f)
        xv,yv = np.linspace(-1.3,1.3,100),np.linspace(-1.3,1.3,100)
        X,Y = np.meshgrid(xv,yv)
        fig = plt.figure()
        ax = fig.gca()
        pg = ax.contour(X,Y,gn(X,Y),0)
        ax.contour(X,Y,fn(X,Y),[-2,-1,0,1,2])
        ax.set_aspect('equal')
```



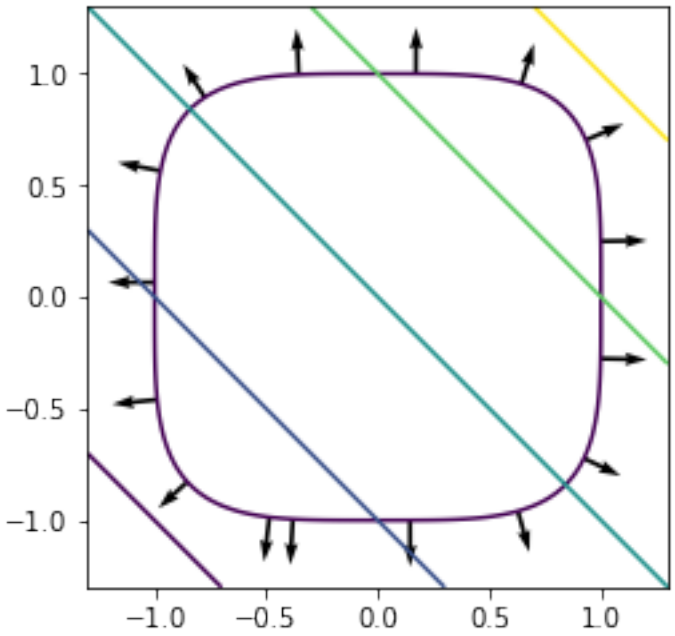

```
In [4]: grad_g = Matrix(2,1,(g.diff(x),g.diff(y)))
        grad_f = Matrix(2,1,(f.diff(x),f.diff(y)))

In [5]: grad_gn = lambdify((x,y),grad_g)
        grad_fn = lambdify((x,y),grad_f)

In [6]: fig = plt.figure()
        ax = fig.gca()
        pg = ax.contour(X,Y,gn(X,Y),0)
        ax.contour(X,Y,fn(X,Y),[-2,-1,0,1,2])
        GF = grad_fn(X[::5,::5],Y[::5,::5])
        ax.quiver(X[::5,::5],Y[::5,::5],GF[0],GF[1],angles='xy')
        ax.set_aspect('equal')
```




```
In [7]: SG = np.array((pg.allsegs)[0][0])
        XG, YG = SG[::20,0], SG[::20,1]
        XG**4+YG**4
```

```
Out[7]: array([ 0.99986689,  0.99995981,  0.99950674,  0.99898692,  0.99903615,
                0.99918032,  0.99900014,  0.99898292,  0.99945686,  0.99959936,
                0.99897277,  0.99896993,  0.99921457,  0.99960604,  0.99898485,
                0.99904591])
```

```
In [8]: fig = plt.figure()
        ax = fig.gca()
        pg = ax.contour(X,Y,gn(X,Y),0)
        ax.contour(X,Y,fn(X,Y),[-2,-1,0,1,2])
        GG = grad_gn(XG,YG)
        ax.quiver(XG,YG,GG[0],GG[1],angles='xy',scale=50)
        ax.set_aspect('equal')
```



In [9]:
```
GLF = grad_f-mu*grad_g
```

In [10]:
```
M = solve([GLF,g])
M
```

Out[10]:

$$\left[\left\{\mu : -\frac{2^{\frac{3}{4}}}{4},\quad x : -\frac{2^{\frac{3}{4}}}{2},\quad y : -\frac{2^{\frac{3}{4}}}{2}\right\},\quad \left\{\mu : \frac{2^{\frac{3}{4}}}{4},\quad x : \frac{2^{\frac{3}{4}}}{2},\quad y : \frac{2^{\frac{3}{4}}}{2}\right\},\quad \left\{\mu : \frac{\sqrt{2}}{8}\left(-1+i\right),\quad x : -\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}+\frac{\sqrt{}}{4}\right.\right.$$

In [11]:
```
rsol = []
for sol in M:
    if (im(sol[mu])==0) & (im(sol[x])==0) & (im(sol[y])==0):
        rsol.append(sol)
rsol
```

Out[11]:

$$\left[\left\{\mu : -\frac{2^{\frac{3}{4}}}{4},\quad x : -\frac{2^{\frac{3}{4}}}{2},\quad y : -\frac{2^{\frac{3}{4}}}{2}\right\},\quad \left\{\mu : \frac{2^{\frac{3}{4}}}{4},\quad x : \frac{2^{\frac{3}{4}}}{2},\quad y : \frac{2^{\frac{3}{4}}}{2}\right\}\right]$$

In [12]:
```
g.subs(rsol[0])
```

Out[12]:

$$0$$

In [13]:
```
g.subs(rsol[1])
```

Out[13]:

$$0$$

In [14]:
```
H = Matrix(2,2,[ (f-mu*g).diff(x1,x2) for x1 in [x,y] for x2 in [x,y]])
H
```

Out[14]:

$$\begin{bmatrix} -12\mu x^2 & 0 \\ 0 & -12\mu y^2 \end{bmatrix}$$

In [15]:
```
H.subs(rsol[0])
```

Out[15]:

$$\begin{bmatrix} 3\sqrt[4]{2} & 0 \\ 0 & 3\sqrt[4]{2} \end{bmatrix}$$

Also haben wir hier ein Minimum (Achtung: Fuer das Minimum muss nur ein Teil der Hessematrix positiv definit sein -> Optimierung

In [16]:
```
H.subs(rsol[1])
```

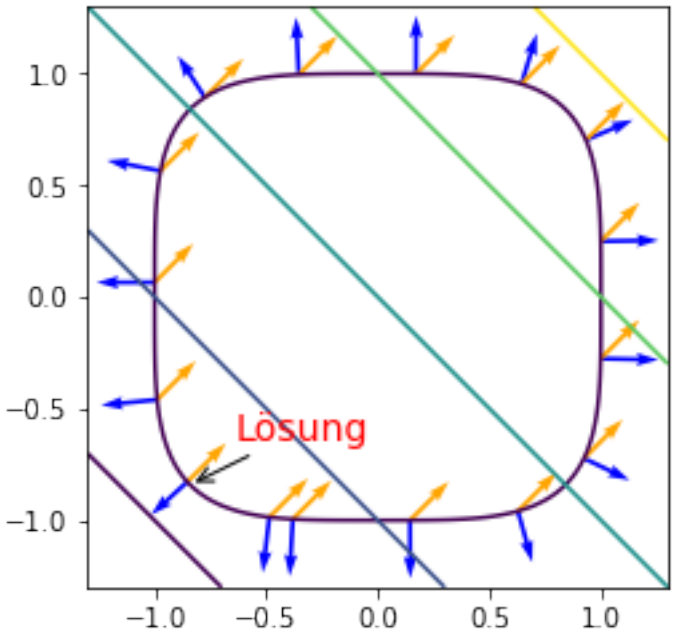
Out[16]:

$$\begin{bmatrix} -3\sqrt[4]{2} & 0 \\ 0 & -3\sqrt[4]{2} \end{bmatrix}$$

In [17]:
```
xx=rsol[0][x]
yy=rsol[0][y]
```

In [18]:
```
fig = plt.figure()
ax = fig.gca()
pg = ax.contour(X,Y,gn(X,Y),0)
ax.contour(X,Y,fn(X,Y),[-2,-1,0,1,2])
GG = grad_gn(XG,YG)
GF = grad_fn(XG,YG)
ax.quiver(XG,YG,GG[0],GG[1],angles='xy',scale=40,color='blue')
ax.quiver(XG,YG,GF[0],GF[1],angles='xy',scale=15,color='orange')
ax.set_aspect('equal')
ax.annotate('Lösung',(xx,yy),(xx+.2,yy+.2),\
            arrowprops={'arrowstyle':'->'},color='red',fontsize=14)
```

Out[18]:
```
<matplotlib.text.Annotation at 0x7fa6435563c8>
```



In [19]: xx

Out[19]:

$$-\frac{2^{\frac{3}{4}}}{2}$$