CompAna 2013/2014 - Klausur

Aufgabe 1

```
> restart:
> with(VectorCalculus):
> BasisFormat(false):
```

a)

```
> f := 1/(1+x^2)*exp(-x^2+1);
```

$$f := \frac{e^{-x^2+1}}{x^2+1} \qquad (1.1.1)$$

```
> f1 := diff(f,x);
```

$$f1 := -\frac{2\,e^{-x^2+1}\,x}{(x^2+1)^2} - \frac{2\,x\,e^{-x^2+1}}{x^2+1} \qquad (1.1.2)$$

```
> normal(f1);
```

$$-\frac{2\,e^{-x^2+1}\,x\,(x^2+2)}{(x^2+1)^2} \qquad (1.1.3)$$

```
> f2 := diff(f,x$2);
```

$$f2 := \frac{8\,e^{-x^2+1}\,x^2}{(x^2+1)^3} + \frac{8\,x^2\,e^{-x^2+1}}{(x^2+1)^2} - \frac{2\,e^{-x^2+1}}{(x^2+1)^2} - \frac{2\,e^{-x^2+1}}{x^2+1} + \frac{4\,x^2\,e^{-x^2+1}}{x^2+1} \qquad (1.1.4)$$

b)

```
> g := sin(x+y)^2+cos(y+z)^2;
```

$$g := \sin(x+y)^2 + \cos(y+z)^2 \qquad (1.2.1)$$

```
> g1 := Gradient(g,[x,y,z]);
```

$$g1 := \begin{bmatrix} 2\sin(x+y)\cos(x+y) \\ 2\sin(x+y)\cos(x+y) - 2\cos(y+z)\sin(y+z) \\ -2\cos(y+z)\sin(y+z) \end{bmatrix} \qquad (1.2.2)$$

```
> g2 := Hessian(g,[x,y,z]);
```

$$g2 := [[2\cos(x+y)^2 - 2\sin(x+y)^2, 2\cos(x+y)^2 - 2\sin(x+y)^2, 0], [2\cos(x+y)^2 - 2\sin(x+y)^2, 2\cos(x+y)^2 - 2\sin(x+y)^2 + 2\sin(y+z)^2 - 2\cos(y+z)^2, 2\sin(y+z)^2 - 2\cos(y+z)^2], [0, 2\sin(y+z)^2 - 2\cos(y+z)^2, 2\sin(y+z)^2 - 2\cos(y+z)^2]] \qquad (1.2.3)$$

```
> H1 := combine(g2);
  H2 := simplify(g2);
```

$H1 :=$

$$[[2\cos(2x+2y), 2\cos(2x+2y), 0],$$
$$[2\cos(2x+2y), 2\cos(2x+2y)-2\cos(2y+2z), -2\cos(2y+2z)],$$
$$[0, -2\cos(2y+2z), -2\cos(2y+2z)]]$$

$$H2 := \begin{bmatrix} 4\cos(x+y)^2-2 & 4\cos(x+y)^2-2 & 0 \\ 4\cos(x+y)^2-2 & 4\cos(x+y)^2-4\cos(y+z)^2 & 2-4\cos(y+z)^2 \\ 0 & 2-4\cos(y+z)^2 & 2-4\cos(y+z)^2 \end{bmatrix} \quad \textbf{(1.2.4)}$$

Aufgabe 2

```
> restart:
```

a)

```
> f := exp(-x^4+x^2);
```

$$f := e^{-x^4+x^2} \quad \textbf{(2.1.1)}$$

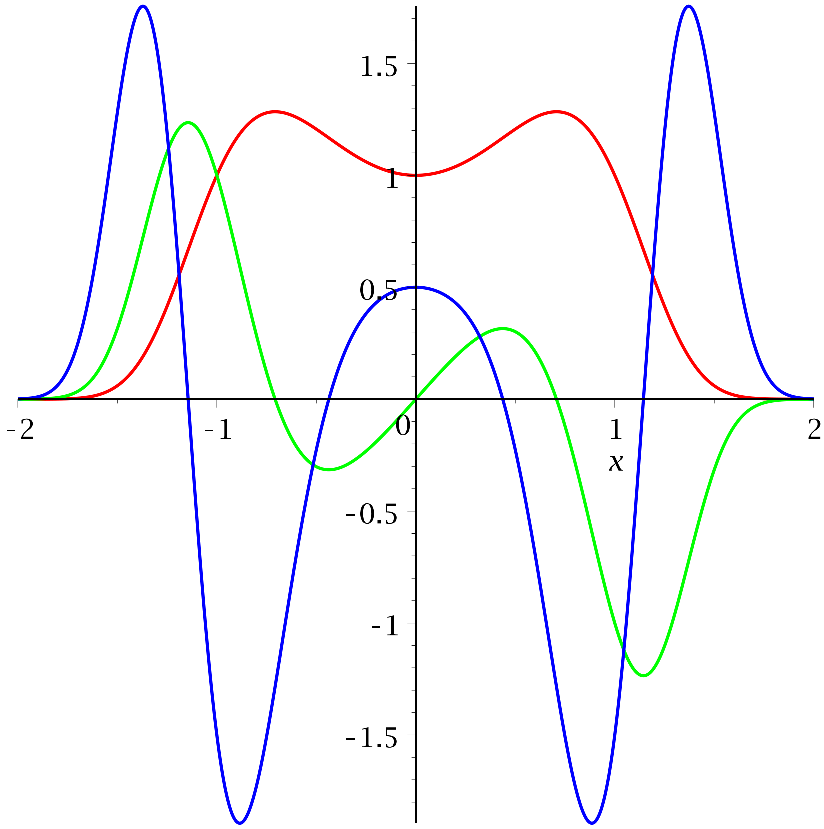
```
> f1 := diff(f,x);
```

$$f1 := (-4x^3+2x)\, e^{-x^4+x^2} \quad \textbf{(2.1.2)}$$

```
> f2 := simplify(diff(f,x$2));
```

$$f2 := 2\, e^{-x^2(x-1)(x+1)} (8x^6-8x^4-4x^2+1) \quad \textbf{(2.1.3)}$$

```
> plot([f,(1/2)*f1,(1/4)*f2],x=-2..2,colour=[red,green,blue],
  numpoints=1000);
```



b)

```
> # kritische Punkte
> K := solve(f1=0,x);
```

$$K := 0, \frac{1}{2}\sqrt{2}, -\frac{1}{2}\sqrt{2} \quad \textbf{(2.2.1)}$$

```
> # Extremalstellen untersuchen
> simplify(subs(x=K[1],f2));
```

$$2 \quad \textbf{(2.2.2)}$$

```
> eval(f2,x=K[1]);
```

$$2 \quad \textbf{(2.2.3)}$$

```
> # K[1]=0 lokale Minimalstelle
> simplify(subs(x=K[2],f2));
```

$$-4\,e^{\frac{1}{4}} \quad \textbf{(2.2.4)}$$

```
> simplify(eval(f2,x=K[2]));
```

(2.2.5)

$$-4\,e^{\frac{1}{4}} \quad (2.2.5)$$

```
> # K[2] lokale Maximalstelle
> simplify(subs(x=K[3],f2));
```

$$-4\,e^{\frac{1}{4}} \quad (2.2.6)$$

```
> simplify(eval(f2,x=K[3]));
```

$$-4\,e^{\frac{1}{4}} \quad (2.2.7)$$

```
> # K[3] lokale Maximalstelle
```

c)

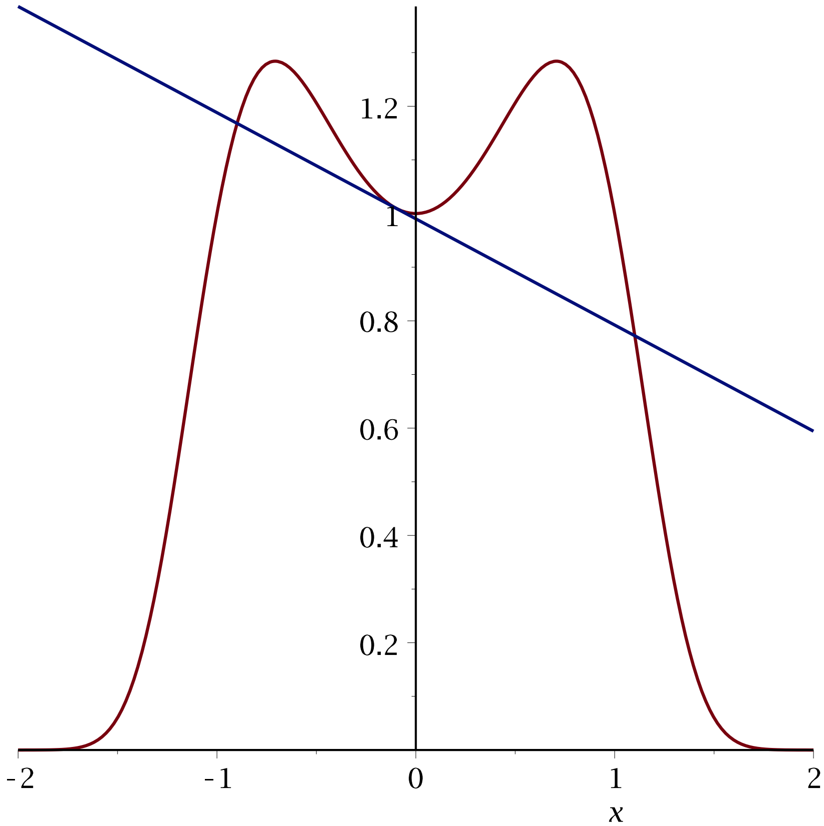
```
> T := simplify(subs(x=-1/10,f)+(x+1/10)*subs(x=-1/10,f1));
```

$$T := -\frac{1}{2500}\,e^{\frac{99}{10000}}\,(-2451+490\,x) \quad (2.3.1)$$

```
> t := simplify(subs(x=-0.1,f)+(x+0.1)*subs(x=-0.1,f1));
```

$$t := 0.9901541633 - 0.1979500367\,x \quad (2.3.2)$$

```
> plot([f,T],x=-2..2);
```




```
> solve(f=T,x,AllSolutions = true);
```
Warning, solutions may have been lost

$$RootOf\left(490\,e^{\frac{99}{10000}}\,\_Z - 2451\,e^{\frac{99}{10000}} + 2500\,e^{-\_Z^4+\_Z^2}\right) \quad \mathbf{(2.3.3)}$$

```
> s1 := fsolve(f=T,x=-2..2);
```
$$s1 := 1.101439469 \quad \mathbf{(2.3.4)}$$

```
> s2 := fsolve(f=T,x,avoid={x=s1});
```
$$s2 := -0.8987054875 \quad \mathbf{(2.3.5)}$$

```
> s3 := fsolve(f=T,x,avoid={x=s1,x=s2});
```
$$s3 := 5.002040816 \quad \mathbf{(2.3.6)}$$

```
> s4 := fsolve(f=T,x=-1..1,avoid={x=s1,x=s2,x=s3});
```
$$s4 := -0.1000000000 \quad \mathbf{(2.3.7)}$$

```
> fsolve(f=T,x,avoid={x=s1,x=s2,x=s3,x=s4});
```
$$fsolve\left(e^{-x^4+x^2} = -\frac{1}{2500}\,e^{\frac{99}{10000}}\,(-2451+490\,x),\ x,\ avoid = \{x = \quad \mathbf{(2.3.8)}$$
$$-0.8987054875,\ x = -0.1000000000,\ x = 1.101439469,\ x$$

$= 5.002040816\})$

```
> # somit keine weiteren Schnittpunkte
```

Aufgabe 3

```
> restart:
> with(plots):
```

a)

```
> a :=-6-4*I;
```

$$a := -6-4\,I \tag{3.1.1}$$

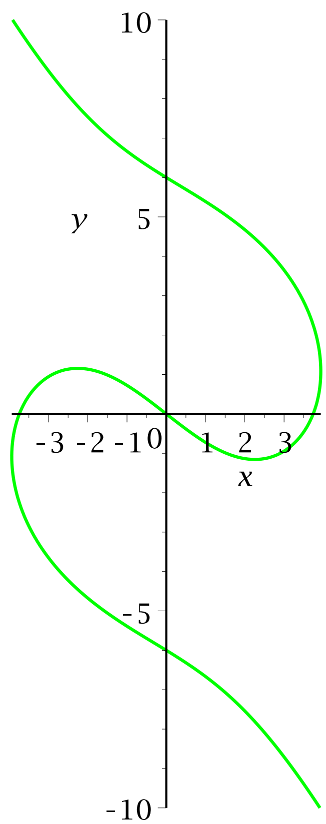
```
> b := 2+2*I;
```

$$b := 2+2\,I \tag{3.1.2}$$

```
> GL := ((x+I*y+a)/(x+I*y-a))*((x+I*y+b)/(x+I*y-b));
```

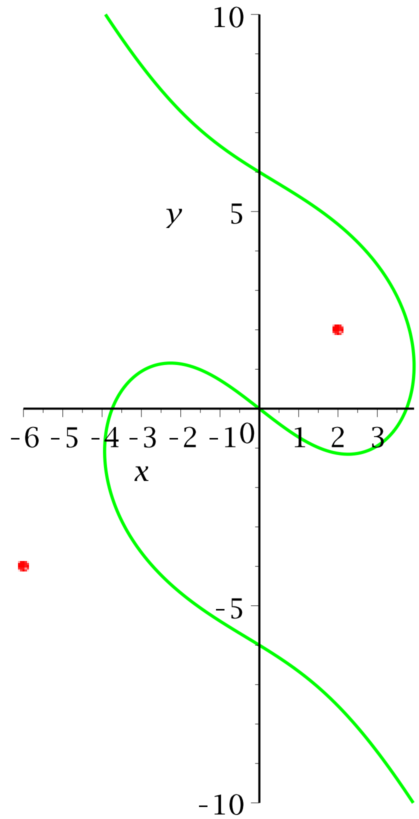
$$GL := \frac{(x+I\,y-6-4\,I)\,(x+I\,y+2+2\,I)}{(x+I\,y+6+4\,I)\,(x+I\,y-2-2\,I)} \tag{3.1.3}$$

```
> implicitplot(abs(GL)=1,x=-10..10,y=-10..10,numpoints=100000,
  scaling=constrained,color=green);
```



b)

```
> P1 := implicitplot(abs(GL)=1,x=-10..10,y=-10..10,numpoints=
  100000,scaling=constrained,color=green):
> P2 := pointplot([-6,-4],symbol=solidcircle,symbolsize=10,
  color=red):
> P3 := pointplot([2,2],symbol=solidcircle,symbolsize=10,
  color=red):
> display(P1,P2,P3);
```



Aufgabe 4

```
> restart:
> with(plots):
> f := exp(-x^2/4)*x;
```

$$f := e^{-\frac{1}{4}x^2}\,x \qquad \textbf{(4.1)}$$

```
> L[0] := 1; L[1] := -x+1;
```

$$L_0 := 1$$

(4.2)

$$L_1 := -x+1 \quad (4.2)$$

```
> for j from 2 to 12 do;
> L[j] := simplify( ((2*j-1-x)/j)*L[j-1]-((j-1)/j)*L[j-2] );
  od;
```

$$L_2 := \frac{1}{2}x^2 - 2x + 1$$

$$L_3 := \frac{3}{2}x^2 - 3x + 1 - \frac{1}{6}x^3$$

$$L_4 := 3x^2 - 4x + 1 - \frac{2}{3}x^3 + \frac{1}{24}x^4$$

$$L_5 := 5x^2 - 5x + 1 - \frac{5}{3}x^3 + \frac{5}{24}x^4 - \frac{1}{120}x^5$$

$$L_6 := \frac{15}{2}x^2 - 6x + 1 - \frac{10}{3}x^3 + \frac{5}{8}x^4 - \frac{1}{20}x^5 + \frac{1}{720}x^6$$

$$L_7 := \frac{21}{2}x^2 - 7x + 1 - \frac{35}{6}x^3 + \frac{35}{24}x^4 - \frac{7}{40}x^5 + \frac{7}{720}x^6 - \frac{1}{5040}x^7$$

$$L_8 := 14x^2 - 8x + 1 - \frac{28}{3}x^3 + \frac{35}{12}x^4 - \frac{7}{15}x^5 + \frac{7}{180}x^6 - \frac{1}{630}x^7 + \frac{1}{40320}x^8$$

$$L_9 := 18x^2 - 9x + 1 - 14x^3 + \frac{21}{4}x^4 - \frac{21}{20}x^5 + \frac{7}{60}x^6 - \frac{1}{140}x^7 + \frac{1}{4480}x^8 - \frac{1}{362880}x^9$$

$$L_{10} := \frac{45}{2}x^2 - 10x + 1 - 20x^3 + \frac{35}{4}x^4 - \frac{21}{10}x^5 + \frac{7}{24}x^6 - \frac{1}{42}x^7 + \frac{1}{896}x^8 - \frac{1}{36288}x^9 + \frac{1}{3628800}x^{10}$$

$$L_{11} := \frac{55}{2}x^2 - 11x + 1 - \frac{55}{2}x^3 + \frac{55}{4}x^4 - \frac{77}{20}x^5 + \frac{77}{120}x^6 - \frac{11}{168}x^7 + \frac{11}{2688}x^8 - \frac{11}{72576}x^9 + \frac{11}{3628800}x^{10} - \frac{1}{39916800}x^{11}$$

$$L_{12} := 33x^2 - 12x + 1 - \frac{110}{3}x^3 + \frac{165}{8}x^4 - \frac{33}{5}x^5 + \frac{77}{60}x^6 - \frac{11}{70}x^7 + \frac{11}{896}x^8 - \frac{11}{18144}x^9 + \frac{11}{604800}x^{10} - \frac{1}{3326400}x^{11} + \frac{1}{479001600}x^{12} \quad (4.3)$$

```
> for k from 0 to 12 do
      c[k] := simplify(int((f*L[k]*exp(-x)),x=0..infinity));
  od;
```

$$c_0 := 2\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - 2\sqrt{\pi}\,\mathrm{e} + 2$$

$$c_1 := 8\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - 8\sqrt{\pi}\,\mathrm{e} + 6$$

$$c_2 := 24\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - 24\sqrt{\pi}\,\mathrm{e} + 18$$

$$c_3 := \frac{142}{3} + \frac{188}{3}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - \frac{188}{3}\sqrt{\pi}\,\mathrm{e}$$

$$c_4 := \frac{340}{3} + \frac{449}{3}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - \frac{449}{3}\sqrt{\pi}\,\mathrm{e}$$

$$c_5 := 254 + \frac{1676}{5}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - \frac{1676}{5}\sqrt{\pi}\,\mathrm{e}$$

$$c_6 := \frac{4874}{9} + \frac{32156}{45}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - \frac{32156}{45}\sqrt{\pi}\,\mathrm{e}$$

$$c_7 := \frac{69914}{63} + \frac{461246}{315}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - \frac{461246}{315}\sqrt{\pi}\,\mathrm{e}$$

$$c_8 := \frac{30813}{14} + \frac{406569}{140}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - \frac{406569}{140}\sqrt{\pi}\,\mathrm{e}$$

$$c_9 := \frac{2407532}{567} + \frac{15883433}{2835}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - \frac{15883433}{2835}\sqrt{\pi}\,\mathrm{e}$$

$$c_{10} := \frac{22677292}{2835} + \frac{149611363}{14175}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - \frac{149611363}{14175}\sqrt{\pi}\,\mathrm{e}$$

$$c_{11} := \frac{153418931}{10395} + \frac{2024336893}{103950}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - \frac{2024336893}{103950}\sqrt{\pi}\,\mathrm{e}$$

$$c_{12} := \frac{10004750993}{374220} + \frac{132011049529}{3742200}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - \frac{132011049529}{3742200}\sqrt{\pi}\,\mathrm{e} \qquad \textbf{(4.4)}$$

```
> # C := seq(simplify(int((f*L[k]*exp(-x)),x=0..infinity)),k=0..12);
```

```
> N := [4,9,12]; k := 'k':
```

$$N := [4, 9, 12] \qquad \textbf{(4.5)}$$

```
> for n in N do
      g[n] := simplify(sum(c[k]*L[k],k=0..n));
  od;
```

$$g_4 := \frac{560}{3} + \frac{449}{72}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^4 + 555\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^2 - \frac{2528}{3}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x - \frac{992}{9}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^3 - 555\sqrt{\pi}\,\mathrm{e}\,x^2 + \frac{2528}{3}\sqrt{\pi}\,\mathrm{e}\,x + \frac{992}{9}\sqrt{\pi}\,\mathrm{e}\,x^3 - \frac{449}{72}\sqrt{\pi}\,\mathrm{e}\,x^4 - \frac{1912}{3}x + 420x^2 - \frac{751}{9}x^3 + \frac{85}{18}x^4 - \frac{739}{3}\sqrt{\pi}\,\mathrm{e} + \frac{739}{3}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)$$

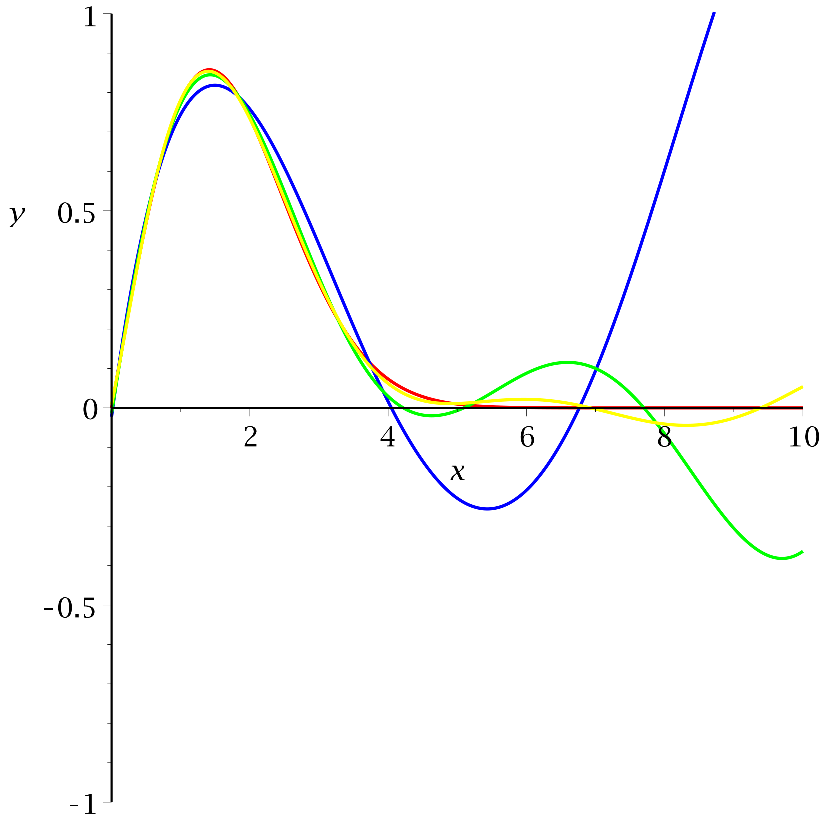
$$g_9 := \frac{9683209}{1134} + \frac{3565661}{79380}\sqrt{\pi}\,\mathrm{e}\,x^7 - \frac{9598979}{7257600}\sqrt{\pi}\,\mathrm{e}\,x^8 + \frac{15883433}{1028764800}\sqrt{\pi}\,\mathrm{e}\,x^9 + \frac{20338433}{2700}\sqrt{\pi}\,\mathrm{e}\,x^5 - \frac{75991301}{97200}\sqrt{\pi}\,\mathrm{e}\,x^6 + \frac{9598979}{7257600}\sqrt{\pi}$$

$$\mathrm{e}\,\mathrm{erf}(1)\,x^8 - \frac{15883433}{1028764800}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^9 - \frac{20338433}{2700}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^5 + \frac{75991301}{97200}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^6 - \frac{3565661}{79380}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^7 + \frac{87570787}{2160}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^4 + \frac{20723131}{126}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^2 - \frac{4082047}{45}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x - \frac{47439154}{405}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^3 - \frac{20723131}{126}\sqrt{\pi}\,\mathrm{e}\,x^2 + \frac{4082047}{45}\sqrt{\pi}\,\mathrm{e}\,x + \frac{47439154}{405}\sqrt{\pi}\,\mathrm{e}\,x^3 - \frac{87570787}{2160}\sqrt{\pi}\,\mathrm{e}\,x^4 - \frac{127768577}{11340}\sqrt{\pi}\,\mathrm{e} + \frac{127768577}{11340}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) - \frac{618724}{9}x + \frac{7852735}{63}x^2 - \frac{7190590}{81}x^3 + \frac{6636779}{216}x^4 - \frac{1541401}{270}x^5 + \frac{5759197}{9720}x^6 - \frac{1080931}{31752}x^7 + \frac{727483}{725760}x^8 - \frac{601883}{51438240}x^9$$

$$g_{12} := \frac{21716694023}{374220} + \frac{8630253763}{778003380000}\sqrt{\pi}\,\mathrm{e}\,x^{11} - \frac{132011049529}{179251978752000 0}\sqrt{\pi}\,\mathrm{e}\,x^{12} - \frac{2954194211}{4199040000}\sqrt{\pi}\,\mathrm{e}\,x^{10} + \frac{10589390984}{1488375}\sqrt{\pi}\,\mathrm{e}\,x^7 - \frac{6411876883}{12192768}\sqrt{\pi}\,\mathrm{e}\,x^8 + \frac{3803017963}{154314720}\sqrt{\pi}\,\mathrm{e}\,x^9 + \frac{23920068713}{70875}\sqrt{\pi}\,\mathrm{e}\,x^5 - \frac{179705534413}{2916000}\sqrt{\pi}\,\mathrm{e}\,x^6 + \frac{2954194211}{4199040000}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^{10} - \frac{8630253763}{778003380000}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^{11} + \frac{132011049529}{179251978752000 0}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^{12} + \frac{6411876883}{12192768}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^8 - \frac{3803017963}{154314720}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^9 - \frac{23920068713}{70875}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^5 + \frac{179705534413}{2916000}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^6 - \frac{10589390984}{1488375}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^7 + \frac{40941510745}{36288}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^4 + \frac{238322019559}{113400}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^2 - \frac{130008626284}{155925}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x - \frac{1572620348}{729}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1)\,x^3 - \frac{238322019559}{113400}\sqrt{\pi}\,\mathrm{e}\,x^2 + \frac{130008626284}{155925}\sqrt{\pi}\,\mathrm{e}\,x + \frac{1572620348}{729}\sqrt{\pi}\,\mathrm{e}\,x^3 - \frac{40941510745}{36288}\sqrt{\pi}\,\mathrm{e}\,x^4 - \frac{286548207919}{3742200}\sqrt{\pi}\,\mathrm{e} + \frac{286548207919}{3742200}\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}(1) \quad \textbf{(4.6)}$$

$$- \frac{19705956496}{31185} x + \frac{18061767503}{11340} x^2 - \frac{1191845821}{729} x^3$$
$$+ \frac{15514223125}{18144} x^4 - \frac{7251344509}{28350} x^5 + \frac{13619386421}{291600} x^6$$
$$- \frac{12840650623}{2381400} x^7 + \frac{2429691991}{6096384} x^8 - \frac{288220193}{15431472} x^9$$
$$+ \frac{223890187}{419904000} x^{10} - \frac{5232503893}{622402704000} x^{11} + \frac{10004750993}{179251978752000} x^{12}$$

```
> plot([f,g[4],g[9],g[12]],x=0..10,y=-1..1,colour=[red,blue,
  green,yellow],numpoints=1000);
```




```
> seq(print('n'=n,seq( evalf( abs(subs(x=a,f)-subs(x=a,g[n])) ),
  a=1..5)),n in N);
```

$n = 4, 0.0341291, 0.022113274, 0.098049, 0.057179, 0.2388493$

$n = 9, 0.007613, 0.00766, 0.01518, 0.043911, 0.01646$

$n = 12, 0.0025, 0.0020, 0.00634, 0.0114, 0.00149$ **(4.7)**

Aufgabe 5

```
> restart:
> Dgl := (1+y(x))*diff(y(x),x) = sin(x)*sqrt(y(x));
```

$$Dgl := (1+y(x))\left(\frac{d}{dx}y(x)\right) = \sin(x)\sqrt{y(x)} \quad (5.1)$$

a)

```
> # Loesung mit Anfangsbedingungen
> Lsg1 := rhs(dsolve({Dgl,y(0)=1},y(x)));
```

$$Lsg1 := \left(\frac{1}{2}\left(22-6\cos(x)+2\sqrt{137-66\cos(x)+9\cos(x)^2}\right)^{1/3} - \frac{2}{\left(22-6\cos(x)+2\sqrt{137-66\cos(x)+9\cos(x)^2}\right)^{1/3}}\right)^2 \quad (5.1.1)$$

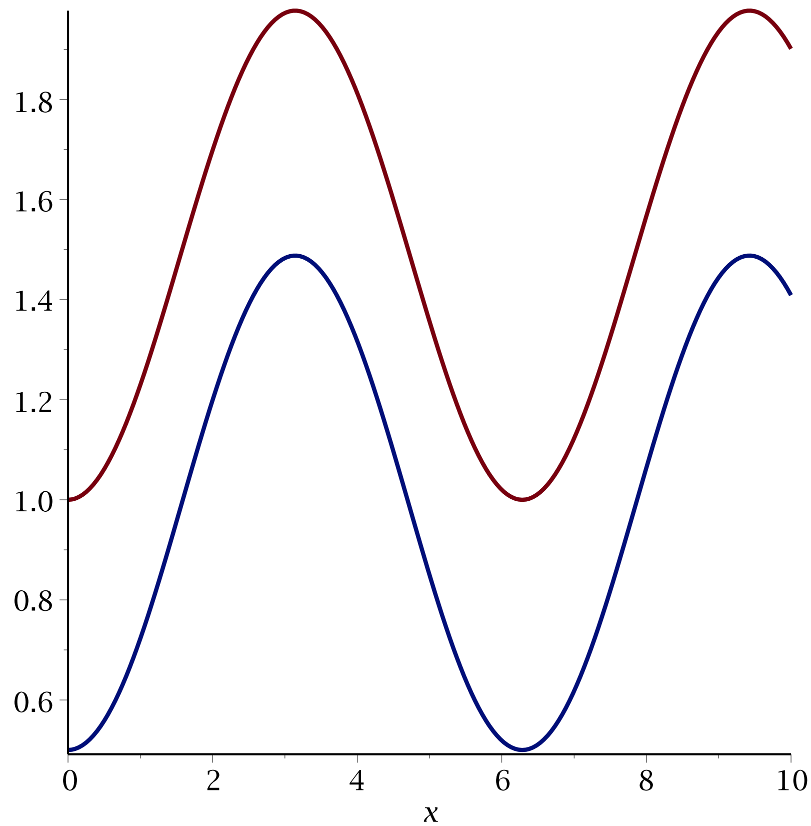
```
> Lsg2 := rhs(dsolve({Dgl,y(0)=1/2},y(x)));
```

$$Lsg2 := \left(\frac{1}{2}\left(6+7\sqrt{2}-6\cos(x) + \sqrt{198+84\sqrt{2}-72\cos(x)-84\sqrt{2}\cos(x)+36\cos(x)^2}\right)^{1/3} \quad (5.1.2)$$

$$\cos(x)^2\Big)^{1/3}\Big)^2$$

b)

```
> P1 := plot([Lsg1,Lsg2],x=0..10,thickness=2):
> P1;
```




```
> v := isolate(Dgl,diff(y(x),x));
```

$$v := \frac{d}{dx}\,y(x) = \frac{\sin(x)\,\sqrt{y(x)}}{1+y(x)} \quad \textbf{(5.2.1)}$$

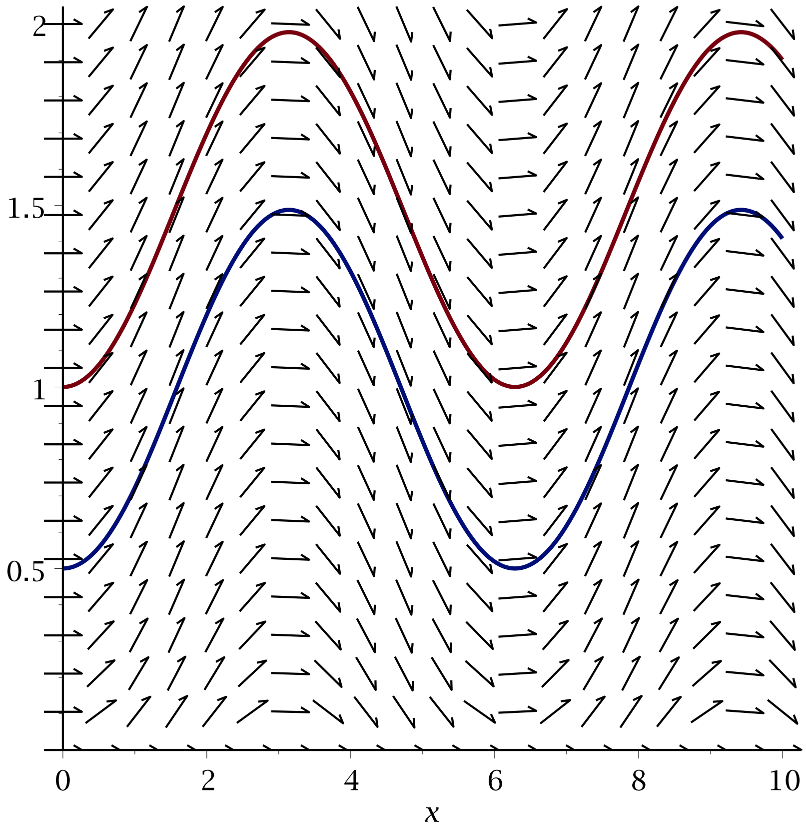
```
> # oder
> # solve(Dgl,diff(y(x),x));
> vf := <1,rhs(v)>;
```

$$vf := \begin{bmatrix} 1 \\ \frac{\sin(x)\,\sqrt{y(x)}}{1+y(x)} \end{bmatrix} \quad \textbf{(5.2.2)}$$

```
> N:=sqrt(vf[1]^2+vf[2]^2):
> w := vf/N:
> with(plots):
> P2 := fieldplot(w,x=0..10,y=0..2,numpoints=1000);
```

$$P2 := PLOT(\ldots) \quad \textbf{(5.2.3)}$$

```
> display(P1, P2);
```


2
1.5
1
0.5
0
2
4
6
8
10
x

>