Loesungen Probeklausur

Aufgabe 1

```
> with(IntegrationTools);
```

$[Change, Combine, Expand, Flip, GetIntegrand, GetOptions, GetParts, GetRange, GetVariable, Parts, Split, StripOptions]$ **(1.1)**

a)

```
> a:= (1+exp(-x))/(1-exp(-x));
```

$$a := \frac{1+e^{-x}}{1-e^{-x}}$$ **(1.1.1)**

```
> A:= Int(a,x);
```

$$A := \int \frac{1+e^{-x}}{1-e^{-x}}\,dx$$ **(1.1.2)**

```
> A1:=Change(A,y=exp(-x));
```

$$A1 := \int \frac{1+y}{(-1+y)\,y}\,dy$$ **(1.1.3)**

```
> GetIntegrand(A1);
```

$$\frac{1+y}{(-1+y)\,y}$$ **(1.1.4)**

```
> convert((1.1.4),parfrac);
```

$$-\frac{1}{y}+\frac{2}{-1+y}$$ **(1.1.5)**

```
> a1:=int(op(1,(1.1.5)),y);
```

$$a1 := -\ln(y)$$ **(1.1.6)**

```
> a2:=int(op(2,(1.1.5)),y);
```

$$a2 := 2\ln(-1+y)$$ **(1.1.7)**

```
> eval(a1+a2,y=exp(-x));
```

$$-\ln(e^{-x})+2\ln(-1+e^{-x})$$ **(1.1.8)**

```
> int(a,x); #Test
```

$$-\ln(e^{-x})+2\ln(-1+e^{-x})$$ **(1.1.9)**

```
> #Alternativ
> value(A1);
```

$$-\ln(y)+2\ln(-1+y)$$ **(1.1.10)**

```
> eval((1.1.10),y=exp(-x));
```

$$-\ln(e^{-x})+2\ln(-1+e^{-x})$$ **(1.1.11)**

b)

```
> b:=2*arctan(x)/(1+x^2);
```

(1.2.1)

$$b := \frac{2\arctan(x)}{1+x^2} \qquad (1.2.1)$$

```
> B:=Int(b,x);
```

$$B := \int \frac{2\arctan(x)}{1+x^2}\,dx \qquad (1.2.2)$$

```
> Parts(B,arctan(x));
```

$$2\arctan(x)^2 - \left(\int \frac{2\arctan(x)}{1+x^2}\,dx\right) \qquad (1.2.3)$$

```
> solve((1.2.3)=B,B);
```

$$\arctan(x)^2 \qquad (1.2.4)$$

```
> int(b,x); #Test
```

$$\arctan(x)^2 \qquad (1.2.5)$$

```
> # Alternativ
> Change(B,y=arctan(x));
```

$$\int 2\,y\,dy \qquad (1.2.6)$$

```
> value((1.2.6));
```

$$y^2 \qquad (1.2.7)$$

```
> eval((1.2.7),y=arctan(x));
```

$$\arctan(x)^2 \qquad (1.2.8)$$

c)

```
> c := sin(x)*x^2;
```

$$c := \sin(x)\,x^2 \qquad (1.3.1)$$

```
> C:= Int(c,x);
```

$$C := \int \sin(x)\,x^2\,dx \qquad (1.3.2)$$

```
> Parts(C,x^2);
```

$$-\cos(x)\,x^2 - \left(\int (-2\cos(x)\,x)\,dx\right) \qquad (1.3.3)$$

```
> Parts((1.3.3),x);
```

$$-\cos(x)\,x^2 + 2\sin(x)\,x + \int (-2\sin(x))\,dx \qquad (1.3.4)$$

```
> value((1.3.4));
```

$$-\cos(x)\,x^2 + 2\sin(x)\,x + 2\cos(x) \qquad (1.3.5)$$

```
> int(c,x); # Test
```

$$-\cos(x)\,x^2 + 2\sin(x)\,x + 2\cos(x) \qquad (1.3.6)$$

d)

```
> d:=sqrt(1+x^2);
```

$$d := \sqrt{1+x^2} \qquad (1.4.1)$$

```
> DD:=Int(d,x);
```

$$DD := \int \sqrt{1+x^2}\,\mathrm{d}x \qquad \textbf{(1.4.2)}$$

```
> simplify(Parts(DD,1));
```

$$\frac{1}{2}\,x\sqrt{1+x^2}+\frac{1}{2}\,\mathrm{arcsinh}(x) \qquad \textbf{(1.4.3)}$$

```
> int(d,x);# Test
```

$$\frac{1}{2}\,x\sqrt{1+x^2}+\frac{1}{2}\,\mathrm{arcsinh}(x) \qquad \textbf{(1.4.4)}$$

```
> # Alternativ ?
> Change(DD,y=1+x^2);
```

$$\int \frac{1}{2}\,\frac{\sqrt{y}}{\sqrt{-1+y}}\,\mathrm{d}y \qquad \textbf{(1.4.5)}$$

```
> Parts((1.4.5),sqrt(y));
```

$$\sqrt{y}\,\sqrt{-1+y}-\left(\int \frac{1}{2}\,\frac{\sqrt{-1+y}}{\sqrt{y}}\,\mathrm{d}y\right) \qquad \textbf{(1.4.6)}$$

```
> simplify(Parts((1.4.6),1));
```

$$-\frac{1}{4}\,\frac{1}{\sqrt{-1+y}\,\sqrt{y}}\left(-2\,y^2+2\,y+\sqrt{(-1+y)\,y}\,\ln(2)-\sqrt{(-1+y)\,y}\,\ln\left(-1+2\,y+2\sqrt{(-1+y)\,y}\right)\right) \qquad \textbf{(1.4.7)}$$

```
> eval((1.4.7),y=1+x^2);
```

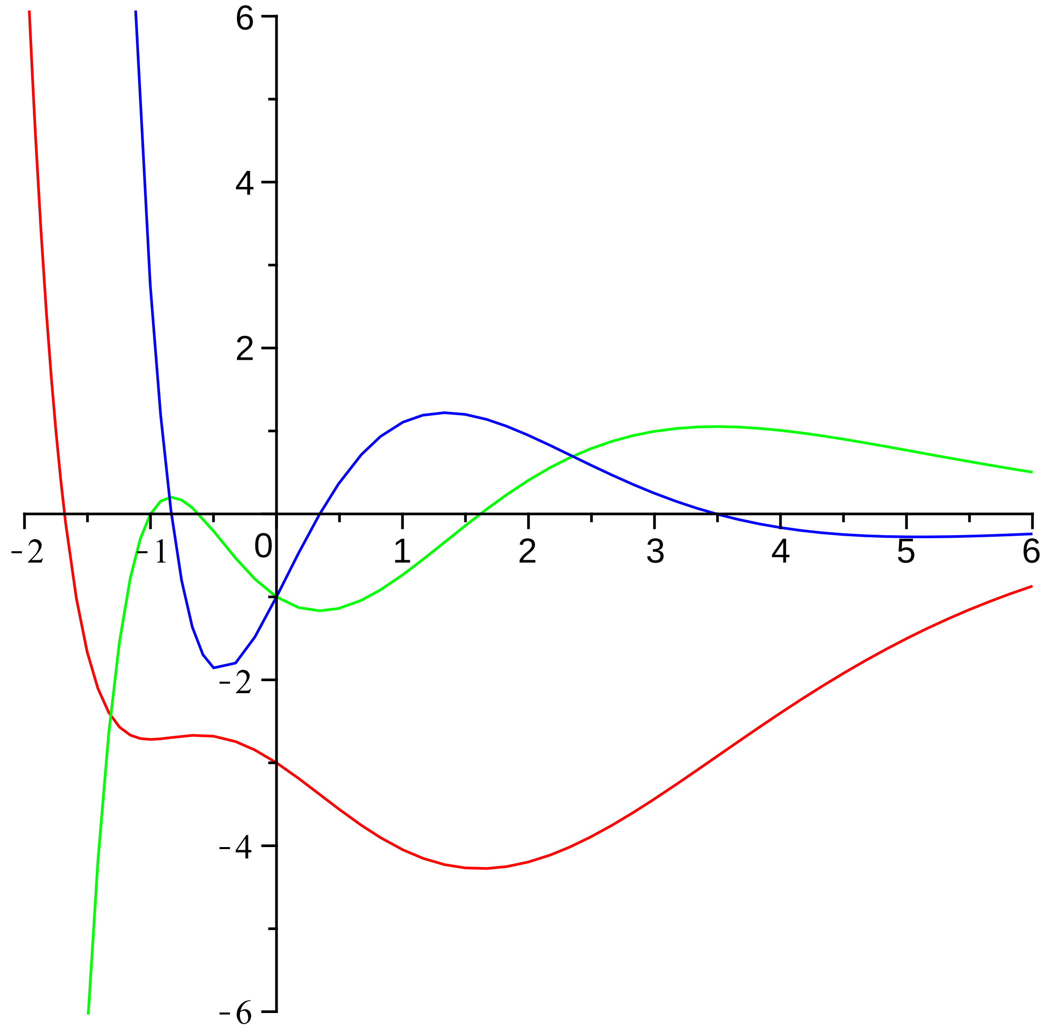
$$-\frac{1}{4}\,\frac{1}{\sqrt{x^2}\,\sqrt{1+x^2}}\left(-2\,(1+x^2)^2+2+2\,x^2+\sqrt{x^2\,(1+x^2)}\,\ln(2)-\sqrt{x^2\,(1+x^2)}\,\ln\left(1+2\,x^2+2\sqrt{x^2\,(1+x^2)}\right)\right) \qquad \textbf{(1.4.8)}$$

Aufgabe 2

```
> restart
Warning, inserted missing semicolon at end of statement
> f := x -> -(x^3+3*x^2+4*x+3)*exp(-x);
```

$$f := x \rightarrow -\left(x^3+3\,x^2+4\,x+3\right)\,\mathrm{e}^{-x} \qquad \textbf{(2.1)}$$

```
> plot([f,D(f),D(D(f))],-2..6,-6..6,color=[red,green,blue]);
```




```
> Extrema := solve(D(f)(x)=0,x);
```

$$Extrema := -1, \frac{1}{2} + \frac{1}{2}\sqrt{5}, \frac{1}{2} - \frac{1}{2}\sqrt{5} \qquad (2.2)$$

```
> evalf(f(Extrema[1]));
```

$$-2.718281828 \qquad (2.3)$$

```
> evalf(f(Extrema[2]));
```

$$-4.275549807 \qquad (2.4)$$

```
> evalf(f(Extrema[3]));
```

$$-2.667320735 \qquad (2.5)$$

```
> evalf(f(-1));
```

$$-2.718281828 \qquad (2.6)$$

```
> evalf(f(2));
```

$$-4.195393779 \qquad (2.7)$$

```
> # daher Maximum bei -1 und Minimum bei Extrema[2]
> for k from 1 to 3 do evalf(D(D(f))(Extrema[k])); od;
```

2.718281828

1.160799067

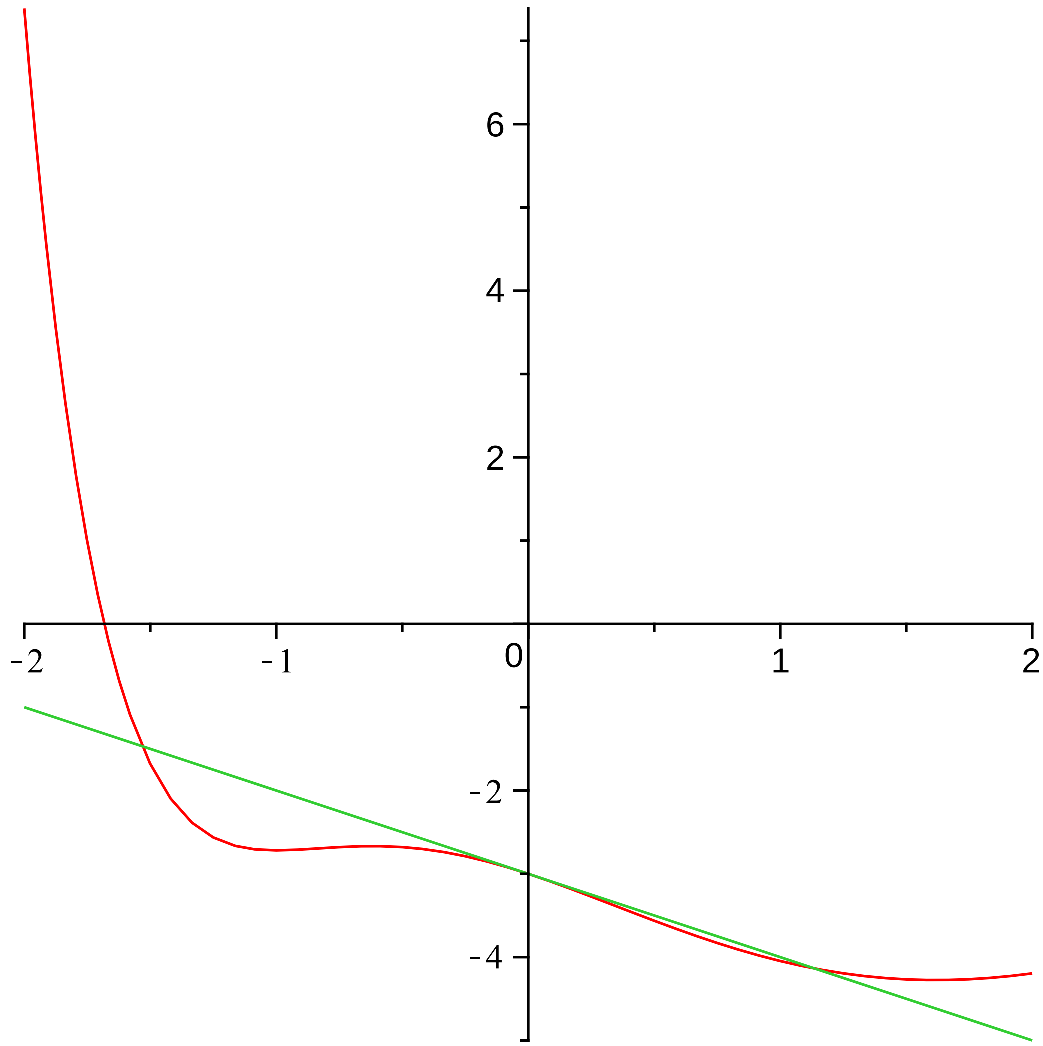
(2.8)

$$-1.584595702 \quad (2.8)$$

```
> # daher Minima bei Extrema[1] und Extrema[2], und ein Maximum
bei Extrema[3]
> g := x -> f(0)+ x*D(f)(0);
```

$$g := x \to f(0) + x\,D(f)(0) \quad (2.9)$$

```
> plot([f,g],-2..2);
```




```
> GL := g(x)=f(x);
```

$$GL := -3 - x = -\left(x^3 + 3\,x^2 + 4\,x + 3\right)\,e^{-x} \quad (2.10)$$

```
> L1 := fsolve(GL,x);
```

$$L1 := 0. \quad (2.11)$$

```
> L2 := fsolve(GL,x,avoid={x=L1});
```

$$L2 := -1.532145884 \quad (2.12)$$

```
> L3 := fsolve(GL,x,avoid={x=L1,x=L2});
```

$$L3 := 1.136613580 \quad (2.13)$$

```
> L4 := fsolve(GL,x,avoid={x=L1,x=L2,x=L3});
```

$$L4 := fsolve\left(-3 - x = -\left(x^3 + 3\,x^2 + 4\,x + 3\right)\,e^{-x}, x, avoid = \{x = -1.532145884, x = 0., x = 1.136613580\}\right) \quad (2.14)$$

Aufgabe 3

```
> restart;
> with(plots);
```

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, graphplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*] **(3.1)**

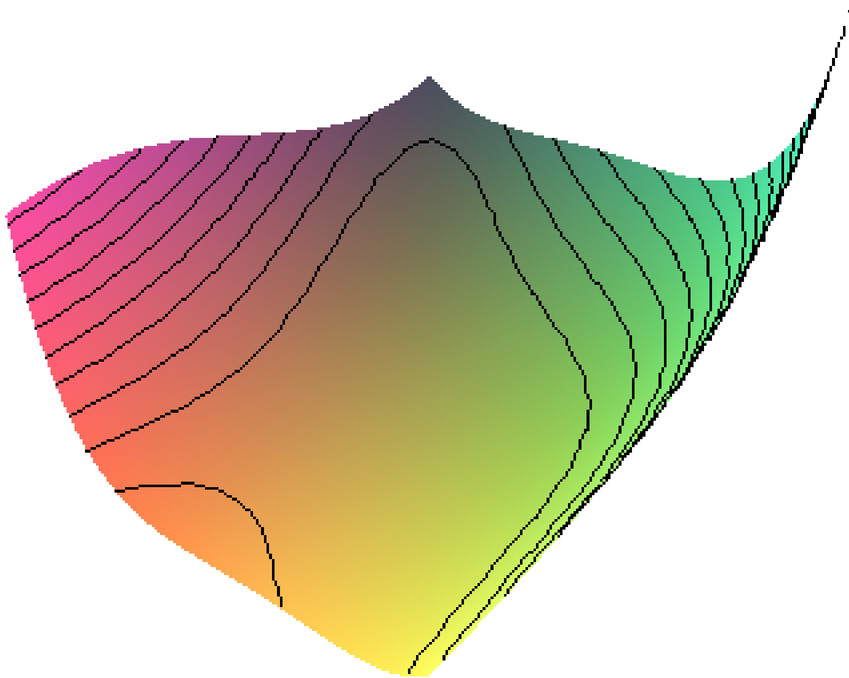
```
> g:= (-x^2-y^3+x)*(x-y)+(x^4+y^4)/6;
```

$$g := \left(-x^2 - y^3 + x\right)\,(x - y) + \frac{1}{6}\,x^4 + \frac{1}{6}\,y^4$$ **(3.2)**

```
> G:=plot3d(g,x=-2..2,y=-2..2,style=patchcontour,contours=20);
```

$$G := PLOT3D(\ldots)$$ **(3.3)**

```
> display(G);
```



```
> e:=(2*x/3)^2+y^2=1;
```

$$e := \frac{4}{9}\,x^2 + y^2 = 1 \tag{3.4}$$

```
> Z:=implicitplot3d((2*x/3)^2+y^2=1,x=-2..2,y=-2..2,z=-10..10,
  transparency=0.5,color=blue,style=patchnogrid);
```

$$Z := PLOT3D(\ldots) \tag{3.5}$$

```
> yy:=solve(e,x);
```

$$yy := \frac{3}{2}\sqrt{1-y^2},\ -\frac{3}{2}\sqrt{1-y^2} \tag{3.6}$$

```
> y1:=yy[1]; y2:=yy[2];
```

$$y1 := \frac{3}{2}\sqrt{1-y^2}$$

$$y2 := -\frac{3}{2}\sqrt{1-y^2} \tag{3.7}$$

```
> z1:=subs(x=y1,g);z2:=subs(x=y2,g);
```

$$z1 := \left(-\frac{9}{4} + \frac{9}{4}\,y^2 - y^3 + \frac{3}{2}\sqrt{1-y^2}\right)\left(\frac{3}{2}\sqrt{1-y^2} - y\right) + \frac{27}{32}\,(1-y^2)^2 + \frac{1}{6}\,y^4$$

$$\tag{3.8}$$

$$z2 := \left(-\frac{9}{4} + \frac{9}{4} y^2 - y^3 - \frac{3}{2}\sqrt{1-y^2}\right)\left(-\frac{3}{2}\sqrt{1-y^2} - y\right) + \frac{27}{32}\left(1-y^2\right)^2 + \frac{1}{6} y^4 \quad (3.8)$$

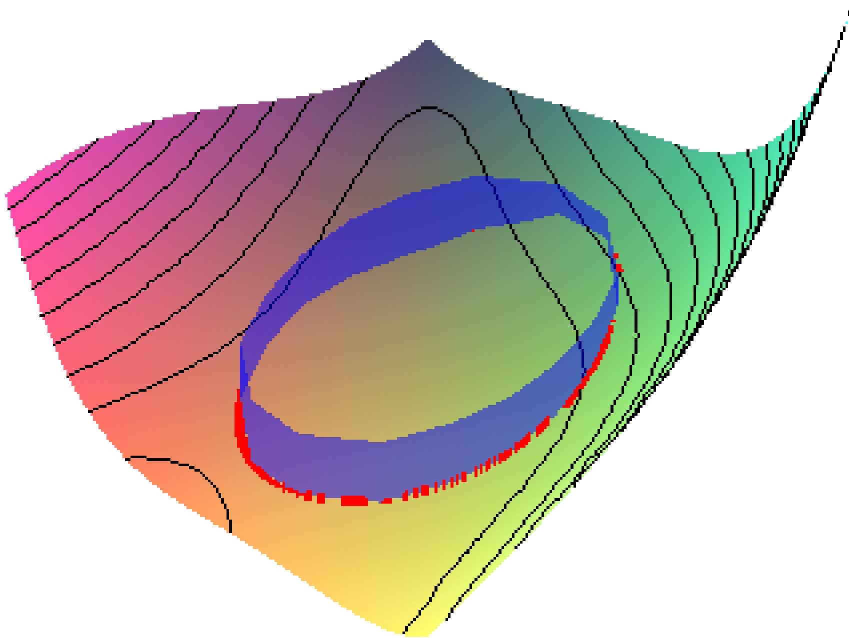
```
> C1:=spacecurve([y1,y,z1],y=-1..1,color=red,thickness=4,
  numpoints=100);
```

$$C1 := PLOT3D(\ldots) \quad (3.9)$$

```
> C2:=spacecurve([y2,y,z2],y=-1..1,color=red,thickness=4,
  numpoints=100);
```

$$C2 := PLOT3D(\ldots) \quad (3.10)$$

```
> display({G,Z,C1,C2});
```



Aufgabe 4

```
> restart;
> f:= exp(-x^2/2);
```

$$f := e^{-\frac{1}{2}x^2} \quad (4.1)$$

```
> for k in [2,12,17,22] do
   TP:=taylor(f,x=0,k);
   P[k]:= convert(TP,polynom);
```

```
> od;
```

$$TP := 1 + O(x^2)$$

$$P_2 := 1$$

$$TP := 1 - \frac{1}{2} x^2 + \frac{1}{8} x^4 - \frac{1}{48} x^6 + \frac{1}{384} x^8 - \frac{1}{3840} x^{10} + O(x^{12})$$

$$P_{12} := 1 - \frac{1}{2} x^2 + \frac{1}{8} x^4 - \frac{1}{48} x^6 + \frac{1}{384} x^8 - \frac{1}{3840} x^{10}$$

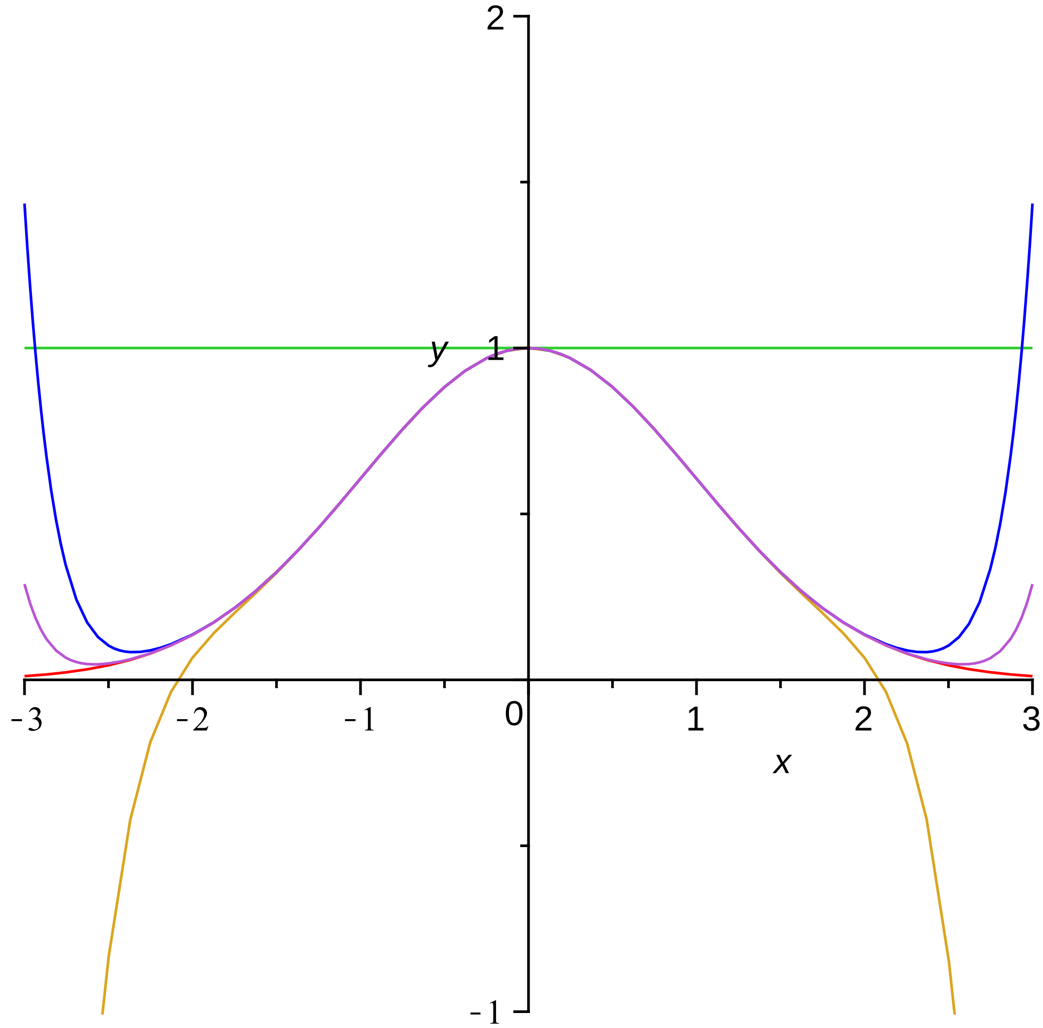
$$TP := 1 - \frac{1}{2} x^2 + \frac{1}{8} x^4 - \frac{1}{48} x^6 + \frac{1}{384} x^8 - \frac{1}{3840} x^{10} + \frac{1}{46080} x^{12} - \frac{1}{645120} x^{14} + \frac{1}{10321920} x^{16} + O(x^{17})$$

$$P_{17} := 1 - \frac{1}{2} x^2 + \frac{1}{8} x^4 - \frac{1}{48} x^6 + \frac{1}{384} x^8 - \frac{1}{3840} x^{10} + \frac{1}{46080} x^{12} - \frac{1}{645120} x^{14} + \frac{1}{10321920} x^{16}$$

$$TP := 1 - \frac{1}{2} x^2 + \frac{1}{8} x^4 - \frac{1}{48} x^6 + \frac{1}{384} x^8 - \frac{1}{3840} x^{10} + \frac{1}{46080} x^{12} - \frac{1}{645120} x^{14} + \frac{1}{10321920} x^{16} - \frac{1}{185794560} x^{18} + \frac{1}{3715891200} x^{20} + O(x^{22})$$

$$P_{22} := 1 - \frac{1}{2} x^2 + \frac{1}{8} x^4 - \frac{1}{48} x^6 + \frac{1}{384} x^8 - \frac{1}{3840} x^{10} + \frac{1}{46080} x^{12} - \frac{1}{645120} x^{14} + \frac{1}{10321920} x^{16} - \frac{1}{185794560} x^{18} + \frac{1}{3715891200} x^{20} \qquad \textbf{(4.2)}$$

```
> plot([f,P[2],P[12],P[17],P[22]],x=-3..3,y=-1..2);
```




```
> print("a=   ",3/2,"     a=    ",1/2,"     a=   ", -2);
  for a in [3/2, 1/2, -2] do
    print("Fehler",seq(evalf(abs(subs(x=a,P[k]-f))),k in [2,12,
  17,22])) ;
  od;
```

"a=   ", $\frac{3}{2}$, "   a=   ", $\frac{1}{2}$, "   a=   ", -2

"Fehler", 0.6753475326, 0.0024196488, 0.0000071432, $8.36\ 10^{-8}$

"Fehler", 0.1175030974, $5.2\ 10^{-9}$, 0., 0.

"Fehler", 0.8646647168, 0.06866861653, 0.0011726533, 0.0000439055 **(4.3)**

```
> for a in [3/2, 1/2, -2] do:
    print("Fehler an Stelle x=",a);
    for k in [2,12,17,22] do
      print("Taylerapproxmation Ordnung:",k ,"Fehler", evalf
  (abs(subs(x=a,P[k]-f)))) ;
    od;
  od;
```

"Fehler an Stelle x=", $\frac{3}{2}$

"Taylerapproxmation Ordnung:", 2, "Fehler", 0.6753475326

"Taylerapproxmation Ordnung:", 12, "Fehler", 0.0024196488

"Taylerapproxmation Ordnung:", 17, "Fehler", 0.0000071432

"Taylerapproxmation Ordnung:", 22, "Fehler", $8.36\ 10^{-8}$

"Fehler an Stelle x=", $\frac{1}{2}$

"Taylerapproxmation Ordnung:", 2, "Fehler", 0.1175030974

"Taylerapproxmation Ordnung:", 12, "Fehler", $5.2\ 10^{-9}$

"Taylerapproxmation Ordnung:", 17, "Fehler", 0.

"Taylerapproxmation Ordnung:", 22, "Fehler", 0.

"Fehler an Stelle x=", -2

"Taylerapproxmation Ordnung:", 2, "Fehler", 0.8646647168

"Taylerapproxmation Ordnung:", 12, "Fehler", 0.06866861653

"Taylerapproxmation Ordnung:", 17, "Fehler", 0.0011726533

"Taylerapproxmation Ordnung:", 22, "Fehler", 0.0000439055 **(4.4)**

```
> for a in [3/2, 1/2, -2] do:
    for k in [2,12,17,22] do
      print(evalf(abs(subs(x=a,P[k]-f)))) ;
    od;
  od;
```

0.6753475326

0.0024196488

0.0000071432

$8.36\ 10^{-8}$

0.1175030974

$5.2\ 10^{-9}$

0.

0.

0.8646647168

0.06866861653

0.0011726533

0.0000439055 **(4.5)**