lektion6

November 20, 2025

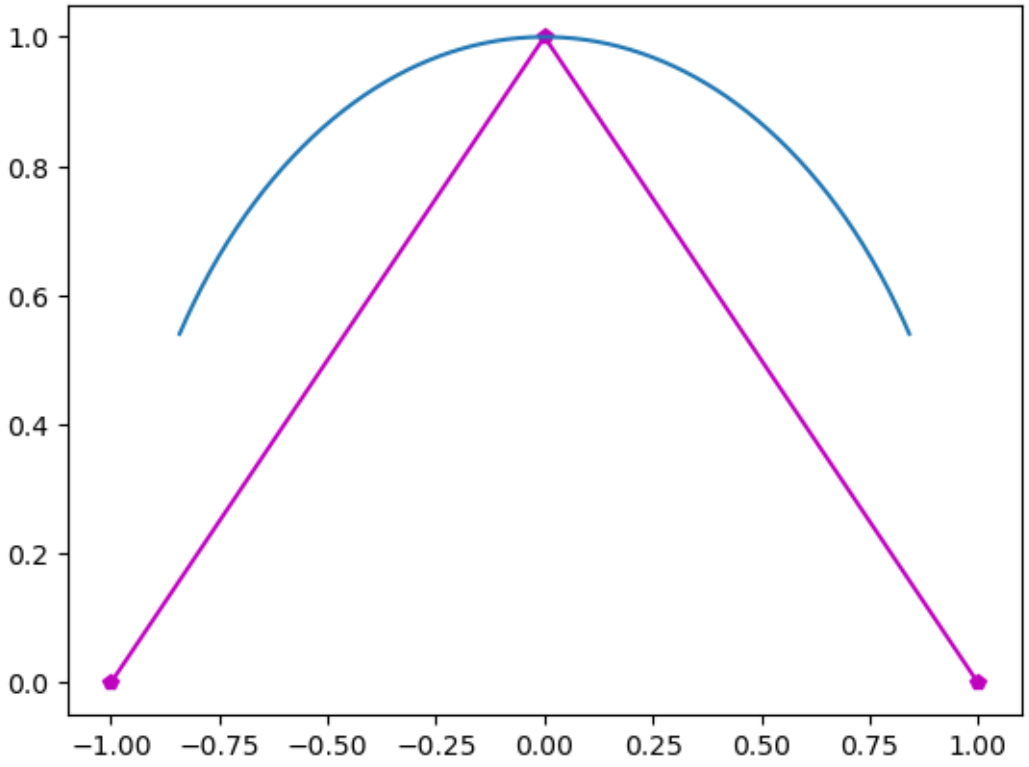
Inhalt

1 Lektion 6

1.1 Wiederholung und Ergänzungen

[1]:
```
#%matplotlib notebook
%matplotlib inline
#%matplotlib qt
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
sp.init_printing()
```

[2]:
```
bild = plt.figure(1)                    # Bild
kors = bild.gca()                       # Koordinatensystem
p1 = kors.plot([-1, 0, 1], [0, 1, 0], 'mp-'  ) # Polygonzug zu Punkten in der␣
 ↪Ebene deren x und y Koordinaten in zwei Listen gegeben sind
xn = np.linspace(-1, 1, 1000)
p2 = kors.plot(np.sin(xn), np.cos(xn)) # zwei Arrays mit x und y Koordinaten␣
 ↪von 1000 Punkten
```



[3]:
```
kors.set_aspect('equal')
p1[0].set_markersize(30) # das geht nur in notebook/qt backend
```

[4]:
```
p1[0].set_markerfacecolor('gold')
p1[0].set_markeredgecolor((1, 0, 0)) # RGB Werte
```

[5]:
```
import matplotlib.colors as mcolors
```

[6]:
```
list(mcolors.BASE_COLORS.keys())
```

[6]:
```
['b', 'g', 'r', 'c', 'm', 'y', 'k', 'w']
```

[7]:
```
mcolors.CSS4_COLORS
```

[7]:
```
{'aliceblue': '#F0F8FF',
 'antiquewhite': '#FAEBD7',
 'aqua': '#00FFFF',
 'aquamarine': '#7FFFD4',
 'azure': '#F0FFFF',
 'beige': '#F5F5DC',
 'bisque': '#FFE4C4',
 'black': '#000000',
 'blanchedalmond': '#FFEBCD',
 'blue': '#0000FF',
 'blueviolet': '#8A2BE2',
 'brown': '#A52A2A',
 'burlywood': '#DEB887',
 'cadetblue': '#5F9EA0',
 'chartreuse': '#7FFF00',
 'chocolate': '#D2691E',
 'coral': '#FF7F50',
 'cornflowerblue': '#6495ED',
 'cornsilk': '#FFF8DC',
 'crimson': '#DC143C',
 'cyan': '#00FFFF',
 'darkblue': '#00008B',
 'darkcyan': '#008B8B',
 'darkgoldenrod': '#B8860B',
 'darkgray': '#A9A9A9',
 'darkgreen': '#006400',
 'darkgrey': '#A9A9A9',
 'darkkhaki': '#BDB76B',
 'darkmagenta': '#8B008B',
 'darkolivegreen': '#556B2F',
 'darkorange': '#FF8C00',
 'darkorchid': '#9932CC',
 'darkred': '#8B0000',
 'darksalmon': '#E9967A',
 'darkseagreen': '#8FBC8F',
 'darkslateblue': '#483D8B',
 'darkslategray': '#2F4F4F',
 'darkslategrey': '#2F4F4F',
 'darkturquoise': '#00CED1',
 'darkviolet': '#9400D3',
 'deeppink': '#FF1493',
 'deepskyblue': '#00BFFF',
 'dimgray': '#696969',
 'dimgrey': '#696969',
 'dodgerblue': '#1E90FF',
 'firebrick': '#B22222',
 'floralwhite': '#FFFAF0',
```

```
'forestgreen': '#228B22',
'fuchsia': '#FF00FF',
'gainsboro': '#DCDCDC',
'ghostwhite': '#F8F8FF',
'gold': '#FFD700',
'goldenrod': '#DAA520',
'gray': '#808080',
'green': '#008000',
'greenyellow': '#ADFF2F',
'grey': '#808080',
'honeydew': '#F0FFF0',
'hotpink': '#FF69B4',
'indianred': '#CD5C5C',
'indigo': '#4B0082',
'ivory': '#FFFFF0',
'khaki': '#F0E68C',
'lavender': '#E6E6FA',
'lavenderblush': '#FFF0F5',
'lawngreen': '#7CFC00',
'lemonchiffon': '#FFFACD',
'lightblue': '#ADD8E6',
'lightcoral': '#F08080',
'lightcyan': '#E0FFFF',
'lightgoldenrodyellow': '#FAFAD2',
'lightgray': '#D3D3D3',
'lightgreen': '#90EE90',
'lightgrey': '#D3D3D3',
'lightpink': '#FFB6C1',
'lightsalmon': '#FFA07A',
'lightseagreen': '#20B2AA',
'lightskyblue': '#87CEFA',
'lightslategray': '#778899',
'lightslategrey': '#778899',
'lightsteelblue': '#B0C4DE',
'lightyellow': '#FFFFE0',
'lime': '#00FF00',
'limegreen': '#32CD32',
'linen': '#FAF0E6',
'magenta': '#FF00FF',
'maroon': '#800000',
'mediumaquamarine': '#66CDAA',
'mediumblue': '#0000CD',
'mediumorchid': '#BA55D3',
'mediumpurple': '#9370DB',
'mediumseagreen': '#3CB371',
'mediumslateblue': '#7B68EE',
'mediumspringgreen': '#00FA9A',
```

```
'mediumturquoise': '#48D1CC',
'mediumvioletred': '#C71585',
'midnightblue': '#191970',
'mintcream': '#F5FFFA',
'mistyrose': '#FFE4E1',
'moccasin': '#FFE4B5',
'navajowhite': '#FFDEAD',
'navy': '#000080',
'oldlace': '#FDF5E6',
'olive': '#808000',
'olivedrab': '#6B8E23',
'orange': '#FFA500',
'orangered': '#FF4500',
'orchid': '#DA70D6',
'palegoldenrod': '#EEE8AA',
'palegreen': '#98FB98',
'paleturquoise': '#AFEEEE',
'palevioletred': '#DB7093',
'papayawhip': '#FFEFD5',
'peachpuff': '#FFDAB9',
'peru': '#CD853F',
'pink': '#FFC0CB',
'plum': '#DDA0DD',
'powderblue': '#B0E0E6',
'purple': '#800080',
'rebeccapurple': '#663399',
'red': '#FF0000',
'rosybrown': '#BC8F8F',
'royalblue': '#4169E1',
'saddlebrown': '#8B4513',
'salmon': '#FA8072',
'sandybrown': '#F4A460',
'seagreen': '#2E8B57',
'seashell': '#FFF5EE',
'sienna': '#A0522D',
'silver': '#C0C0C0',
'skyblue': '#87CEEB',
'slateblue': '#6A5ACD',
'slategray': '#708090',
'slategrey': '#708090',
'snow': '#FFFAFA',
'springgreen': '#00FF7F',
'steelblue': '#4682B4',
'tan': '#D2B48C',
'teal': '#008080',
'thistle': '#D8BFD8',
'tomato': '#FF6347',
```

```
 'turquoise': '#40E0D0',
 'violet': '#EE82EE',
 'wheat': '#F5DEB3',
 'white': '#FFFFFF',
 'whitesmoke': '#F5F5F5',
 'yellow': '#FFFF00',
 'yellowgreen': '#9ACD32'}
```

1.2 Bilder abspeichern

[8]:
```
bild.savefig('einbild2.png')
```

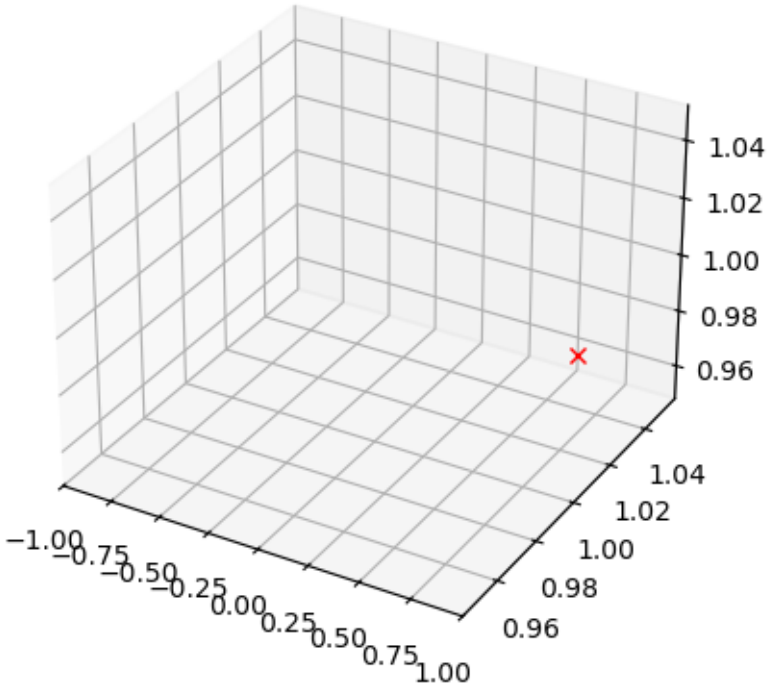
[9]:
```
bild.canvas.get_supported_filetypes()
```

[9]:
```
{'eps': 'Encapsulated Postscript',
 'jpg': 'Joint Photographic Experts Group',
 'jpeg': 'Joint Photographic Experts Group',
 'pdf': 'Portable Document Format',
 'pgf': 'PGF code for LaTeX',
 'png': 'Portable Network Graphics',
 'ps': 'Postscript',
 'raw': 'Raw RGBA bitmap',
 'rgba': 'Raw RGBA bitmap',
 'svg': 'Scalable Vector Graphics',
 'svgz': 'Scalable Vector Graphics',
 'tif': 'Tagged Image File Format',
 'tiff': 'Tagged Image File Format',
 'webp': 'WebP Image Format'}
```

[10]:
```
bild.savefig('einbild.svg', format='svg')
```

1.3 3D Koordinatensysteme

[11]:
```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')  # 3d Koordinatensystem
ax.plot(1, 1, 1, 'rx')
ax.set_xlim3d((-1, 1));
```



1.4 Implizit gegebene Flächen Isoflächen (marching_cube + plot_trisurf)

Alle Punkte $(x, y, z) \in \mathbb{R}^3$ derart, dass

$$f(x, y, z) = 0 \quad \text{für} \quad f : \mathbb{R}^3 \to \mathbb{R}$$

[12]:
```
%matplotlib qt
import matplotlib.pyplot as plt
import numpy as np
from skimage import measure

xn = np.linspace(-2, 2, 100)
dx = xn[1] - xn[0]
X, Y, Z = np.meshgrid(xn, xn, xn)
vol = X**4 + Y**4 + Z**4 + \
      1000 * (X**4 + Y**4) * (X**4 + Z**4) * (Y**4 + Z**4) - 10
#vol = X**2+Y**2+Z**2-4                    # Kugel
#vol = (np.sqrt(X**2+Y**2)-1)**2+Z**2-1/4  # Torus

p0, t0, _, __ = measure.marching_cubes(vol, level=0, spacing=(dx, dx, dx))
# 0-Isofläche beschrieben durch Dreiecksflächen
# p0: Array mit x,y,z Koordinaten der Eckpunte
# t0: Array mit den Eckpunktenummern der Dreiecke
```

```
p0 += xn[0]
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.plot_trisurf(p0[:, 0], p0[:, 1], p0[:, 2],
                triangles=t0,
                color='red',
                alpha=0.5)
```

[12]: <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f47704721b0>

1.5 Nochmal Listen

1.5.1 Zugriff auf Elemente einer Liste (Slicing von Listen)

[13]:
```
a = [1, 2, [3, 4], [5, 6], 7]
a
```

[13]: [1, 2, [3, 4], [5, 6], 7]

[14]:
```
a[0] # erstes Element
```

[14]: 1

[15]:
```
a[-1]  # letztes Element
```

[15]: 7

[16]:
```
a[0:-1] # jedes Element vom Ersten (0) bis Vorletzten (-1)
```

[16]: [1, 2, [3, 4], [5, 6]]

[17]:
```
a[::2]  # jedes zweite Element vom Ersten (0) bis zum Letzten
```

[17]: [1, [3, 4], 7]

a[anfang:ende:increment] 'ende' ist exklusiv

[18]:
```
c = a[0:-1:2]
c
```

[18]: [1, [3, 4]]

[19]:
```
b = a + c #  Zusammenfügen von Listen
b
```

[19]: [1, 2, [3, 4], [5, 6], 7, 1, [3, 4]]

[20]:
```
b[2][0] = 'pi' #Achtung
```

[21]:
```
b
```

[21]:
```
[1, 2, ['pi', 4], [5, 6], 7, 1, ['pi', 4]]
```

[22]:
```
3*c
```

[22]:
```
[1, ['pi', 4], 1, ['pi', 4], 1, ['pi', 4]]
```

1.5.2 Flaches und tiefes kopieren (copy / deepcopy)

[23]:
```
al = [1, 21, [3, 4]]
```

[24]:
```
ac = al.copy()  # flache Kopie
ac[0] = 'hallo'
ac[2][0] = 2225
```

[25]:
```
al
```

[25]:
$$[1,\ 21,\ [2225,\ 4]]$$

[26]:
```
ac
```

[26]:
```
['hallo', 21, [2225, 4]]
```

[27]:
```
from copy import deepcopy
al = [1, 21, [3, 4]]
```

[28]:
```
ad = deepcopy(al)# tiefe Kopie
```

[29]:
```
ad[0] = 'hallo'
ad[2][0] = 2225
```

[30]:
```
ad
```

[30]:
```
['hallo', 21, [2225, 4]]
```

[31]:
```
al
```

[31]:
$$[1,\ 21,\ [3,\ 4]]$$

[32]:
```
b = al + deepcopy(al)
```

[33]:
```
b
```

[33]:
$$[1,\ 21,\ [3,\ 4]\,,\ 1,\ 21,\ [3,\ 4]]$$

[34]:
```
b[2][0]= 331
```

[35]:
```
b
```

[35]:
```
[1, 21, [331, 4], 1, 21, [3, 4]]
```

1.6 Zu den aktuellen Aufgaben

1.6.1 zu Aufgabe 24

[36]:
```
def f (c):  # abschnittweise definierte Funktion
    if c > 1:
        c= 1
    elif c <-1:
        c =-1
    else:
        c = c**3
    return c
```

[37]:
```
f(2.3)
```

[37]:
```
1
```

[38]:
```
f(-2), f(-1), f(0), f(0.5), f(1), f(2)
```

[38]:
```
(-1, -1, 0, 0.125, 1, 1)
```

1.6.2 Funktionen mit mehreren Ausgaben

[39]:
```
def g(c):
    q = c**2
    mc = -c
    return c, q, mc
```

[40]:
```
erg = g(2)
erg
```

[40]:
```
(2, 4, -2)
```

[41]:
```
a, b, c = g(2)
```

[42]:
```
a, c
```

[42]:
```
(2, -2)
```

1.7 Ergänzungen zu Schleifen

1.7.1 enumerate

[43]:
```
for e in [2, 5, 7, 9]:
    print(e)
```

```
2
5
7
9
```

[44]:
```
for i, e in enumerate([2, 5, 7, 9]):
    print(i, e)
```

```
0 2
1 5
2 7
3 9
```

1.7.2 zip (Reißverschluss)

[45]:
```
for e in zip([12, 23, 43, 2 , 3 ], ['a', 'b', 'c', 'd']):
    print(e[0], e[1])
```

```
12 a
23 b
43 c
2 d
```

1.7.3 Erzeugen von Wörterbüchern (dictionary)

[46]:
```
d = {}
for e in zip([12, 23, 43], ['a', 'b', 'c']):
    d |= {e[0] : e[1]}  # inplace 'oder' geht seit Python 3.9
d
```

[46]:
```
{12: 'a', 23: 'b', 43: 'c'}
```

[47]:
```
d = {}
for e in zip([12, 23, 43], ['a', 'b', 'c']):
    d.update({e[0] : e[1]})
d
```

[47]:
```
{12: 'a', 23: 'b', 43: 'c'}
```

[48]:
```
q = { -i: i**2 for i in range(2, 10)} # Wörterbuch der Quadratzahlen
```

[49]:
```
q
```

[49]:
$$\{-9 : 81,\ -8 : 64,\ -7 : 49,\ -6 : 36,\ -5 : 25,\ -4 : 16,\ -3 : 9,\ -2 : 4\}$$

[50]:
```
q[-2]
```

[50]:
$$4$$

1.8 Numerische Auswertung von Integralen

siehe auch Lektion 3 **Wenn sympy nicht mehr weiter weiß**

[51]:
```
import sympy as sp

x = sp.symbols('x')
I1 = sp.Integral(sp.sin(x**2 + sp.sin(x)), (x, 0, 20))
I1
```

[51]:
$$\int_0^{20} \sin\left(x^2 + \sin\left(x\right)\right) dx$$

[52]:
```
sp.plot(I1.args[0], (x, 0, 20))
```

[52]:
```
<sympy.plotting.backends.matplotlibbackend.matplotlib.MatplotlibBackend at
0x7f4784efe840>
```

[53]:
```
I1.doit()  # das ist zu schwierig
```

[53]:
$$\int_0^{20} \sin\left(x^2 + \sin\left(x\right)\right) dx$$

[54]:
```
%%timeit #numerische Auswertung des Integrals -> mehr dazu in Numerik 1
I1.n()
```

```
364 ms ± 26.7 ms per loop (mean ± std. dev. of 7 runs, 1 loop each)
```

[55]:
```
I1.n()
```

[55]:
$$0.54682529389423$$

[56]:
```
I1.n(60)
```

[56]:
$$0.546825293894230355024236014576530667519413898834411146571265$$

[57]:
```
I1.args
```

[57]:
$$\left(\sin\left(x^2 + \sin\left(x\right)\right),\ \left(x,\ 0,\ 20\right)\right)$$

[58]:
```
# Mit numpy/scipy geht das schneller, aber nicht mit belieger Genauigkeit
from scipy import integrate

fn = sp.lambdify(x, I1.args[0])
```

[59]:
```
integrate.quad(fn, 0, 20)
```

[59]:
$$\left(0.546825293894244,\ 1.475908724837 \cdot 10^{-8}\right)$$

[60]:
```
%%timeit
integrate.quad(fn, 0, 20)
```

```
2.84 ms ± 253  s per loop (mean ± std. dev. of 7 runs, 100 loops each)
```