

Blatt 9

Aufgabe 34

```
> restart:
```

```
> f := (5*x^3 - 3*x - 2)/((2 + x)*(3-x));
```

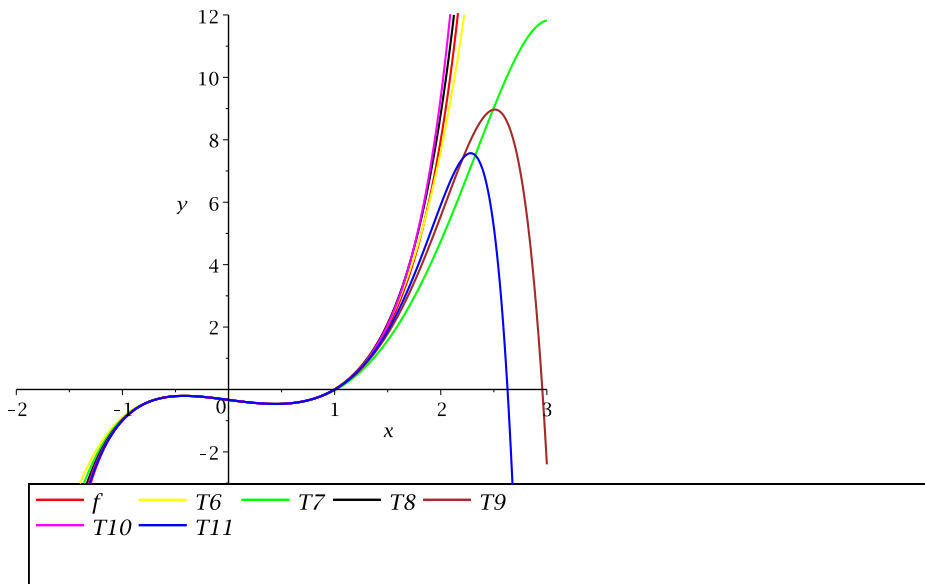
$$f := \frac{5x^3 - 3x - 2}{(2+x)(3-x)}$$

(1.1)

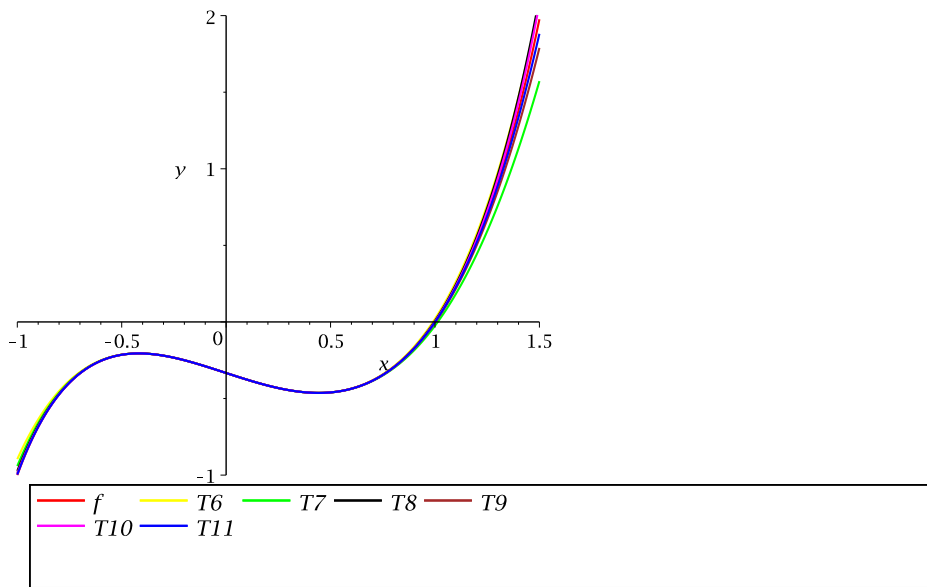
(a)

```
> T := seq(convert(taylor(f, x = 0, k), polynom), k = 1..15):
```

```
> plot([ f, seq(T[j], j = 6..11) ], x = -2..3, y = -3..12, color  
= [ red, yellow, green, black, brown, magenta, blue ],  
numpoints = 2000, legend = [ 'f', 'T6', 'T7', 'T8', 'T9',  
'T10', 'T11' ]);
```



```
> plot([ f, seq(T[j], j = 6..11) ], x = -1..3/2, y = -1..2, color  
= [ red, yellow, green, black, brown, magenta, blue ],  
numpoints = 2000, legend = [ 'f', 'T6', 'T7', 'T8', 'T9',  
'T10', 'T11' ] );
```



(c)

```

> seq(abs(evalf(subs(x = -1/2, f) - subs(x = -1/2, T[j]))), j =
1..15);
0.1190476190, 0.1031746032, 0.1078042328, 0.01328262787,
0.005598728689, 0.001020003552, 0.0003182806580,
0.00006902353616, 0.00001901365543, 4.460451959 10-6,
1.163939973 10-6, 2.828471627 10-7, 7.206809576 10-8,
1.779097309 10-8, 4.485418415 10-9
(1.2)

> seq(abs(evalf(subs(x = 1, f) - subs(x = 1, T[j]))), j = 1..15);
0.3333333333, 0.7777777778, 0.7592592593, 0.003086419753,
0.1260288066, 0.02049039781, 0.02441986740, 0.007485044201,
0.005317485266, 0.002133754911, 0.001241873363,
0.0005626047124, 0.0003007463459, 0.0001438918430,
0.00007410636482
(1.3)

```

▼ Aufgabe 35

```
> restart:
```

```
(a)
```

```
> P := 2, 3, 4, 5;
```

$$P := 2, 3, 4, 5 \quad (2.1)$$

```
> seq(print(Sum((n^p)/(p^n), n = 1..infinity) = sum((n^p)/(p^n),  
n = 1..infinity)), p in P);
```

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n} = 6$$

$$\sum_{n=1}^{\infty} \frac{n^3}{3^n} = \frac{33}{8}$$

$$\sum_{n=1}^{\infty} \frac{n^4}{4^n} = \frac{380}{81}$$

$$\sum_{n=1}^{\infty} \frac{n^5}{5^n} = \frac{3535}{512} \quad (2.2)$$

```
(b)
```

```
> An := (2*n + 1)^2 / ((2*n + 1)^2 + 1),  
(-1)^n * (1 - 1/n),  
(-1)^n * (1 - n/(n-1));
```

$$An := \frac{(2n+1)^2}{(2n+1)^2+1}, (-1)^n \left(1 - \frac{1}{n}\right), (-1)^n \left(1 - \frac{n}{n-1}\right) \quad (2.3)$$

```
> seq(print(Product(an, n = 2..infinity) = evalc(product(an, n =  
2..infinity))), an in An);
```

$$\prod_{n=2}^{\infty} \frac{(2n+1)^2}{(2n+1)^2+1} = \frac{20}{9 \cosh\left(\frac{\pi}{2}\right)}$$

$$\prod_{n=2}^{\infty} (-1)^n \left(1 - \frac{1}{n}\right) = 0$$

$$\prod_{n=2}^{\infty} (-1)^n \left(1 - \frac{n}{n-1}\right) = 0 \quad (2.4)$$

```
> #Product((2*n + 1)^2 / ((2*n + 1)^2 + 1), n = 2..infinity) =  
product((2*n + 1)^2 / ((2*n + 1)^2 + 1), n = 2..infinity);
```

```
> #evalc(%);
```

```
> #Product((-1)^n * (1 - 1/n), n = 2..infinity) = product((-1)^n  
* (1 - 1/n), n = 2..infinity);
```

```
> #Product((-1)^n * (1 - n/(n - 1)), n = 2..infinity) = product(  
(-1)^n * (1 - n/(n - 1)), n = 2..infinity);
```

▼ Aufgabe 36

```
> restart:
```

```

> MWS := proc (f, a, b)
  local s, abl, w, t, p;

  s := (subs(x = b, f) - subs(x = a, f)) / (b - a);
  # oder s := (eval(f, x = b) - subs(f, x = a)) / (b-a);
  abl := diff(f, x);
  w := solve(abl = s, x);

  # Tangente
  t := subs(x = w, f) + subs(x = w, abl) * (x - w);

  p := plot([ f, t ], x = a..b):
  return w, t, p;
end proc;

```

MWS := **proc**(*f*, *a*, *b*)

(3.1)

local *s*, *abl*, *w*, *t*, *p*;

s := (subs(*x* = *b*, *f*) - subs(*x* = *a*, *f*)) / (*b* - *a*);

abl := diff(*f*, *x*);

w := solve(*abl* = *s*, *x*);

t := subs(*x* = *w*, *f*) + subs(*x* = *w*, *abl*) * (*x* - *w*);

p := plot([*f*, *t*], *x* = *a*..*b*);

return *w*, *t*, *p*

end proc

```

> f := x^2 - 3 * x + 5;

```

```

> y, t, p := MWS(f, 1, 4):

```

```

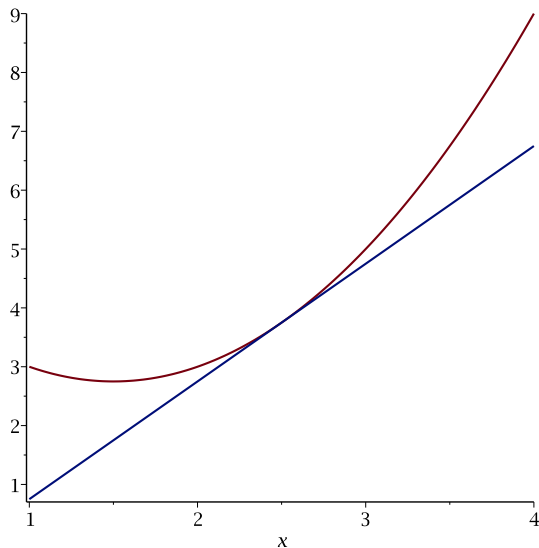
> 'x[0]' = y; 'Tangente(x)' = t; p;

```

$$f := x^2 - 3x + 5$$

$$x_0 = \frac{5}{2}$$

$$\text{Tangente}(x) = -\frac{5}{4} + 2x$$



Aufgabe 37

```
> restart:
```

```
> with(VectorCalculus):
```

```
> k := t -> [
    -10*cos(t) - 2*cos(5*t) + 15*sin(2*t),
    15*cos(2*t) + 10*sin(t) - 2*sin(5*t),
    10*cos(3*t)
];
```

```
k := t ↦ [ -(10 cos(t)) + (-2 cos(5 t)) + (15 sin(2 t)), 15 cos(2 t)
+ (10 sin(t)) + (-2 sin(5 t)), 10 cos(3 t)] (4.1)
```

```
> #k := [-10*cos(t) - 2*cos(5*t) + 15*sin(2*t), 15*cos(2*t) + 10*
sin(t) - 2*sin(5*t), 10*cos(3*t)];
```

```
> kk := diff(k(t), t);
kk := [10 sin(t) + 10 sin(5 t) + 30 cos(2 t), -30 sin(2 t) + 10 cos(t)
- 10 cos(5 t), -30 sin(3 t)] (4.2)
```

```
> B := sqrt(simplify(sum(kk[j]^2, j = 1..3)));
```

(4.3)

$$B \tag{4.3}$$

$$:= 10 (384 \sin(t) \cos(t)^6 - 208 \cos(t)^6 - 480 \sin(t) \cos(t)^4 + 312 \cos(t)^4 + 144 \sin(t) \cos(t)^2 - 117 \cos(t)^2 - 12 \sin(t) + 22)^{1/2}$$

$$\begin{aligned} &> L := \text{int}(B, t = 0..2*Pi); \\ L := & \tag{4.4} \end{aligned}$$

$$\int_0^{2\pi} 10 (384 \sin(t) \cos(t)^6 - 208 \cos(t)^6 - 480 \sin(t) \cos(t)^4 + 312 \cos(t)^4 + 144 \sin(t) \cos(t)^2 - 117 \cos(t)^2 - 12 \sin(t) + 22)^{1/2} dt$$

$$\begin{aligned} &> \text{evalf}(L); \\ & \qquad \qquad \qquad 240.4842902 \tag{4.5} \end{aligned}$$

Aufgabe 38

$$\begin{aligned} &> \text{restart}; \\ &> K := 5; \\ & \qquad \qquad \qquad K := 5 \tag{5.1} \end{aligned}$$

$$\begin{aligned} &> \text{ang} := k \rightarrow \text{Pi}/2 + 2*Pi*k/K; \\ &> \text{vec} := (k, l) \rightarrow [\cos(\text{ang}(k)), \sin(\text{ang}(k))] * l; \\ & \qquad \qquad \qquad \text{ang} := k \mapsto \frac{\pi}{2} + \frac{2\pi k}{K} \\ & \qquad \qquad \qquad \text{vec} := (k, l) \mapsto [\cos(\text{ang}(k)), \sin(\text{ang}(k))] l \tag{5.2} \end{aligned}$$

$$\begin{aligned} &> \# \text{ Punkte auf dem \u00e4u\u00dferen Kreis} \\ &> w1 := [\text{seq}(\text{vec}(k, 1), k = 0 .. K)]; \\ w1 := & \left[[0, 1], \left[-\cos\left(\frac{\pi}{10}\right), \sin\left(\frac{\pi}{10}\right) \right], \left[-\cos\left(\frac{3\pi}{10}\right), -\sin\left(\frac{3\pi}{10}\right) \right], \right. \\ & \left. \left[\cos\left(\frac{3\pi}{10}\right), -\sin\left(\frac{3\pi}{10}\right) \right], \left[\cos\left(\frac{\pi}{10}\right), \sin\left(\frac{\pi}{10}\right) \right], [0, 1] \right] \tag{5.3} \end{aligned}$$

$$\begin{aligned} &> \# \text{ Punkte auf dem inneren Kreis} \\ &> w2 := [\text{seq}(\text{vec}(k + 1/2, 2/5), k = 0 .. K-1)]; \\ w2 := & \left[\left[\frac{2 \cos\left(\frac{3\pi}{10}\right)}{5}, \frac{2 \sin\left(\frac{3\pi}{10}\right)}{5} \right], \left[\frac{2 \cos\left(\frac{\pi}{10}\right)}{5}, \frac{2 \sin\left(\frac{\pi}{10}\right)}{5} \right], \left[0, \right. \right. \\ & \left. \left. -\frac{2}{5} \right], \left[\frac{2 \cos\left(\frac{\pi}{10}\right)}{5}, -\frac{2 \sin\left(\frac{\pi}{10}\right)}{5} \right], \left[\frac{2 \cos\left(\frac{3\pi}{10}\right)}{5}, \frac{2 \sin\left(\frac{3\pi}{10}\right)}{5} \right] \right] \tag{5.4} \end{aligned}$$

$$\begin{aligned} &> \# \text{ Vereinfacht} \\ &> \text{with(ListTools):} \\ &> w := \text{Interleave}(w1, w2); \end{aligned}$$

$$\begin{aligned}
 w := & \left[[0, 1], \left[-\frac{2 \cos\left(\frac{3\pi}{10}\right)}{5}, \frac{2 \sin\left(\frac{3\pi}{10}\right)}{5} \right], \left[-\cos\left(\frac{\pi}{10}\right), \sin\left(\frac{\pi}{10}\right) \right], \right. \\
 & \left. -\frac{2 \cos\left(\frac{\pi}{10}\right)}{5}, -\frac{2 \sin\left(\frac{\pi}{10}\right)}{5} \right], \left[-\cos\left(\frac{3\pi}{10}\right), -\sin\left(\frac{3\pi}{10}\right) \right], \left[0, -\frac{2}{5} \right], \\
 & \left[\cos\left(\frac{3\pi}{10}\right), -\sin\left(\frac{3\pi}{10}\right) \right], \left[\frac{2 \cos\left(\frac{\pi}{10}\right)}{5}, -\frac{2 \sin\left(\frac{\pi}{10}\right)}{5} \right], \left[\cos\left(\frac{\pi}{10}\right), \right. \\
 & \left. \sin\left(\frac{\pi}{10}\right) \right], \left[\frac{2 \cos\left(\frac{3\pi}{10}\right)}{5}, \frac{2 \sin\left(\frac{3\pi}{10}\right)}{5} \right], [0, 1] \right]
 \end{aligned}
 \tag{5.5}$$

> # Von Hand

> w := proc (k) if is(k, even) then w2[k/2] else w1[(k+1)/2] end if; end proc;

> w := seq(w(k), k = 1..2*K+1);

$$\begin{aligned}
 w := & [0, 1], \left[-\frac{2 \cos\left(\frac{3\pi}{10}\right)}{5}, \frac{2 \sin\left(\frac{3\pi}{10}\right)}{5} \right], \left[-\cos\left(\frac{\pi}{10}\right), \sin\left(\frac{\pi}{10}\right) \right], \right. \\
 & \left. -\frac{2 \cos\left(\frac{\pi}{10}\right)}{5}, -\frac{2 \sin\left(\frac{\pi}{10}\right)}{5} \right], \left[-\cos\left(\frac{3\pi}{10}\right), -\sin\left(\frac{3\pi}{10}\right) \right], \left[0, -\frac{2}{5} \right], \\
 & \left[\cos\left(\frac{3\pi}{10}\right), -\sin\left(\frac{3\pi}{10}\right) \right], \left[\frac{2 \cos\left(\frac{\pi}{10}\right)}{5}, -\frac{2 \sin\left(\frac{\pi}{10}\right)}{5} \right], \left[\cos\left(\frac{\pi}{10}\right), \right. \\
 & \left. \sin\left(\frac{\pi}{10}\right) \right], \left[\frac{2 \cos\left(\frac{3\pi}{10}\right)}{5}, \frac{2 \sin\left(\frac{3\pi}{10}\right)}{5} \right], [0, 1]
 \end{aligned}
 \tag{5.6}$$

> plot([w], thickness = 2, scaling = constrained, axes='none');

