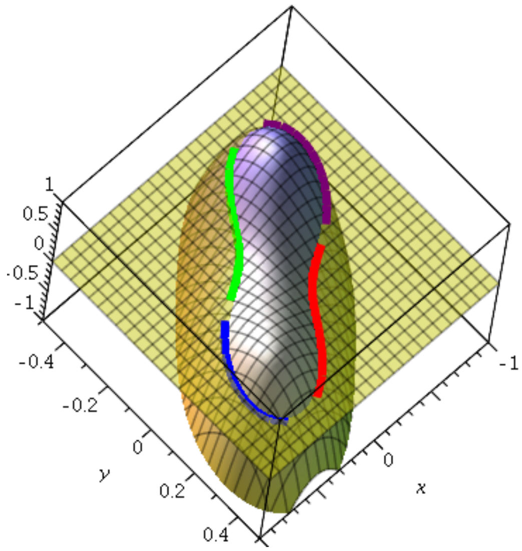
Blatt 7

Aufgabe 26

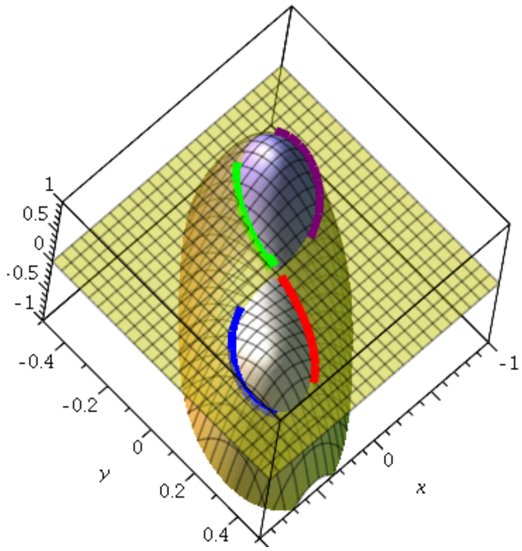
```
> restart:
> with(plots):
> P := (x, y) -> 6*x*y - 3*y^2 - 4*x^4 + 8*x^3*y - 24*x^2*y^2 +
  20*x*y^3 - 25*y^4 - a;
```

$$P := (x, y) \mapsto -4\,x^4 + 8\,x^3\,y - 24\,x^2\,y^2 + 20\,x\,y^3 - 25\,y^4 + 6\,x\,y - 3\,y^2 - a \quad (1.1)$$

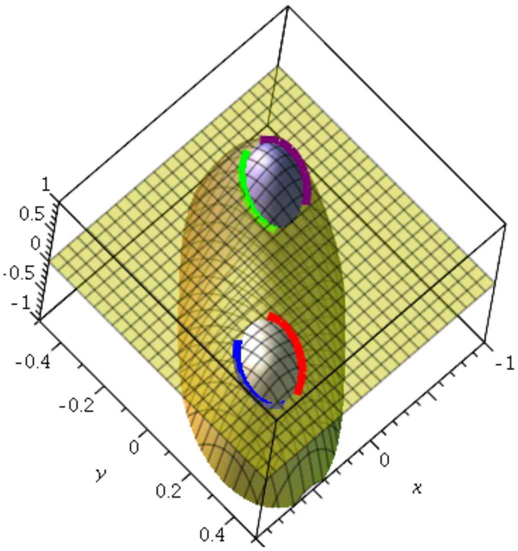
```
> as := [ -1/10, 0, 1/10 ]:
> for kk from 1 to nops(as) do
    solutions := [ allvalues(solve(subs(a = as[kk], P(x, y)) = 0,
  { x,y })) ]:

    # Polynomial and z-plane
    p0 := plot3d(subs(a = as[kk], P(x, y)), x = -1..1, y= -0.5.
  .0.5):
    pz := plot3d([ x, y, 0 ], x = -1..1, y= -0.5..0.5, color =
  yellow, transparency = 0.5):

   # Plot of the zeros
   colors := [ blue, red, green, purple ];
   p := [ seq(0, ll = 1..nops(solutions)) ];
   for ll from 1 to nops(solutions) do
      p[ll] := spacecurve([ rhs(solutions[ll][1]), rhs(solutions
  [ll][2]), 0 ], y = -1..1, color = colors[ll], thickness = 10):
   end do;

   # Show it!
   p := [ op(p), p0, pz ];
   print(display(p, view = [ -1..1, -1/2..1/2, -1..1],
  orientation = [ 48, 22, 4 ]));
  end do:
```



1
0.5
0
-0.5
-1
-0.4
-0.2
0
y
0.2
0.4
-1
0
x



1
0.5
0
-0.5
-1
-0.4
-0.2
0
y
0.2
0.4
-1
0
x



Aufgabe 27

```
> restart:
```

(a)

```
> f := arctan(x) * exp(1 + x^3) * ln(x^2 + 1);
> df  := diff(f, x);
> d2f := diff(f, x$2);
```

$$f := \arctan(x)\, e^{x^3+1} \ln(x^2+1)$$

$$df := \frac{e^{x^3+1}\ln(x^2+1)}{x^2+1} + 3\arctan(x)\, x^2\, e^{x^3+1}\ln(x^2+1) + \frac{2\arctan(x)\, e^{x^3+1}\, x}{x^2+1}$$

$$d2f := -\frac{2\, e^{x^3+1}\ln(x^2+1)\, x}{(x^2+1)^2} + \frac{6\, x^2\, e^{x^3+1}\ln(x^2+1)}{x^2+1} + \frac{4\, e^{x^3+1}\, x}{(x^2+1)^2} + 6\arctan(x)\, x\, e^{x^3+1}\ln(x^2+1) + 9\arctan(x)\, x^4\, e^{x^3+1}\ln(x^2+1) \tag{2.1}$$

$$+\frac{12\arctan(x)\,x^3\,e^{x^3+1}}{x^2+1}+\frac{2\arctan(x)\,e^{x^3+1}}{x^2+1}-\frac{4\arctan(x)\,e^{x^3+1}\,x^2}{(x^2+1)^2}$$

```
> collect(df, [exp(1+x^3),arctan(x),ln(x^2+1)]);
```

$$\left(\left(3x^2\ln(x^2+1)+\frac{2x}{x^2+1}\right)\arctan(x)+\frac{\ln(x^2+1)}{x^2+1}\right)e^{x^3+1} \quad (2.2)$$

```
> collect(d2f, [exp(1+x^3),arctan(x),ln(x^2+1)]);
```

$$\left(\left((9x^4+6x)\ln(x^2+1)-\frac{4x^2}{(x^2+1)^2}+\frac{12x^3}{x^2+1}+\frac{2}{x^2+1}\right)\arctan(x)+\left(-\frac{2x}{(x^2+1)^2}+\frac{6x^2}{x^2+1}\right)\ln(x^2+1)+\frac{4x}{(x^2+1)^2}\right)e^{x^3+1} \quad (2.3)$$

(b)

```
> b := cos(2*arctan(x));
```

$$b := \cos(2\arctan(x)) \quad (2.4)$$

```
> b = simplify(b);
```

$$\cos(2\arctan(x)) = \cos(2\arctan(x)) \quad (2.5)$$

```
> expand(b);
```

$$\frac{2}{x^2+1}-1 \quad (2.6)$$

```
> normal(expand(b));
```

$$-\frac{x^2-1}{x^2+1} \quad (2.7)$$

```
> normal(expand(b), expanded);
```

$$\frac{-x^2+1}{x^2+1} \quad (2.8)$$

```
> normal(simplify(expand(b)));
```

$$-\frac{x^2-1}{x^2+1} \quad (2.9)$$

```
> normal(simplify(expand(b)),expanded);
```

$$\frac{-x^2+1}{x^2+1} \quad (2.10)$$

```
> convert(b, tan);
```

$$\frac{1-\tan(\arctan(x))^2}{1+\tan(\arctan(x))^2} \quad (2.11)$$

```
> simplify(convert(b, tan));
```

$$-\frac{x^2-1}{x^2+1} \quad (2.12)$$

```
> L := trigsubs(b);
```

$$L := \left[\cos(-2\arctan(x)), \cos(\arctan(x))^2 - \sin(\arctan(x))^2, \frac{1}{\sec(2\arctan(x))}, \frac{1-\tan(\arctan(x))^2}{1+\tan(\arctan(x))^2}, \frac{e^{2\,I\arctan(x)}}{2} + \frac{e^{-2\,I\arctan(x)}}{2}, \frac{2}{x^2+1} - 1, -\frac{x^2}{x^2+1} + \frac{1}{x^2+1}\right] \quad \textbf{(2.13)}$$

```
> b = normal(L[7]);
```

$$\cos(2\arctan(x)) = -\frac{x^2-1}{x^2+1} \quad \textbf{(2.14)}$$

Aufgabe 28

```
> restart:
```

(a)

```
> a := y^2;
```

$$a := y^2 \quad \textbf{(3.1)}$$

```
> sqrt(a);
```

$$\sqrt{y^2} \quad \textbf{(3.2)}$$

```
> sqrt(a) assuming y>=0;
```

$$y \quad \textbf{(3.3)}$$

```
> sqrt(a) assuming y<0;
```

$$-y \quad \textbf{(3.4)}$$

(b)

```
> sin(n*Pi/2) assuming n::even;
```

$$0 \quad \textbf{(3.5)}$$

```
> sin(n*Pi/2) assuming n::odd;
```

$$(-1)^{\frac{n}{2}-\frac{1}{2}} \quad \textbf{(3.6)}$$

Aufgabe 29

```
> restart:
> with(LinearAlgebra):
```

(a)

```
> M := <<1,5,3,4>|<3,1,2,0>>;
```

$$M := \begin{bmatrix} 1 & 3 \\ 5 & 1 \\ 3 & 2 \\ 4 & 0 \end{bmatrix} \quad \textbf{(4.1)}$$

```
> N := Transpose(M);
```

$$N := \begin{bmatrix} 1 & 5 & 3 & 4 \\ 3 & 1 & 2 & 0 \end{bmatrix} \quad \textbf{(4.2)}$$

```
> MMt := M . N;
```

$$MMt := \begin{bmatrix} 10 & 8 & 9 & 4 \\ 8 & 26 & 17 & 20 \\ 9 & 17 & 13 & 12 \\ 4 & 20 & 12 & 16 \end{bmatrix} \quad \textbf{(4.3)}$$

```
> MtM := N . M;
```

$$MtM := \begin{bmatrix} 51 & 14 \\ 14 & 14 \end{bmatrix} \quad \textbf{(4.4)}$$

```
> Rank(MMt);
```

2 **(4.5)**

```
> Determinant(MMt);
```

0 **(4.6)**

```
> Rank(MtM);
```

2 **(4.7)**

```
> Determinant(MtM);
```

518 **(4.8)**

(b)

```
> S := SubMatrix(MMt, 2..3, 2..3);
```

$$S := \begin{bmatrix} 26 & 17 \\ 17 & 13 \end{bmatrix} \quad \textbf{(4.9)}$$

```
> (S^2) . (MtM^(-1));
```

$$\begin{bmatrix} \frac{302}{37} & \frac{20303}{518} \\ \frac{205}{37} & \frac{7038}{259} \end{bmatrix} \quad \textbf{(4.10)}$$

(c)

```
> T := MMt + Matrix(<0, 1+2*t, 1-3*t, 0>, shape = diagonal);
```

$$T := \begin{bmatrix} 10 & 8 & 9 & 4 \\ 8 & 27+2t & 17 & 20 \\ 9 & 17 & 14-3t & 12 \\ 4 & 20 & 12 & 16 \end{bmatrix} \quad \textbf{(4.11)}$$

```
> q := Determinant(T);
```

(4.12)

$$q := -864\,t^2 - 144\,t + 144 \quad (4.12)$$

```
> solve(q = 0, t);
```

$$-\frac{1}{2}, \frac{1}{3} \quad (4.13)$$