Blatt 6

Aufgabe 21

```
> restart:
> ns := seq(n, n = 2..12);
```

$$ns := 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 \quad \textbf{(1.1)}$$

```
> p := (x, n) -> x^n - 1;
```

$$p := (x, n) \mapsto x^n - 1 \quad \textbf{(1.2)}$$

```
> for n in ns do
    solve(p(x, n) = 0, x);
  end do;
```

$$1, -1$$

$$1, -\frac{1}{2} - \frac{I\sqrt{3}}{2}, -\frac{1}{2} + \frac{I\sqrt{3}}{2}$$

$$1, -1, I, -I$$

$$1, \frac{\sqrt{5}}{4} - \frac{1}{4} + \frac{I\sqrt{2}\sqrt{5+\sqrt{5}}}{4}, -\frac{\sqrt{5}}{4} - \frac{1}{4} + \frac{I\sqrt{2}\sqrt{5-\sqrt{5}}}{4}, -\frac{\sqrt{5}}{4} - \frac{1}{4} - \frac{I\sqrt{2}\sqrt{5-\sqrt{5}}}{4}, \frac{\sqrt{5}}{4} - \frac{1}{4} - \frac{I\sqrt{2}\sqrt{5+\sqrt{5}}}{4}$$

$$1, -1, \frac{\sqrt{-2-2I\sqrt{3}}}{2}, -\frac{\sqrt{-2-2I\sqrt{3}}}{2}, \frac{\sqrt{-2+2I\sqrt{3}}}{2}, -\frac{\sqrt{-2+2I\sqrt{3}}}{2}$$

$$1, \cos\left(\frac{2\pi}{7}\right) + I\sin\left(\frac{2\pi}{7}\right), -\cos\left(\frac{3\pi}{7}\right) + I\sin\left(\frac{3\pi}{7}\right), -\cos\left(\frac{\pi}{7}\right) + I\sin\left(\frac{\pi}{7}\right), -\cos\left(\frac{\pi}{7}\right) - I\sin\left(\frac{\pi}{7}\right), -\cos\left(\frac{3\pi}{7}\right) - I\sin\left(\frac{3\pi}{7}\right), \cos\left(\frac{2\pi}{7}\right) - I\sin\left(\frac{2\pi}{7}\right)$$

$$1, -1, I, -I, \frac{\sqrt{2}}{2} + \frac{I\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} - \frac{I\sqrt{2}}{2}, \frac{\sqrt{2}}{2} - \frac{I\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} + \frac{I\sqrt{2}}{2}$$

$$1, -\frac{1}{2} - \frac{I\sqrt{3}}{2}, -\frac{1}{2} + \frac{I\sqrt{3}}{2}, \frac{(-4+4I\sqrt{3})^{1/3}}{2}, -\frac{(-4+4I\sqrt{3})^{1/3}}{4} + \frac{I\sqrt{3}(-4+4I\sqrt{3})^{1/3}}{4}, -\frac{(-4+4I\sqrt{3})^{1/3}}{4} - \frac{I\sqrt{3}(-4+4I\sqrt{3})^{1/3}}{4}, \frac{(-4-4I\sqrt{3})^{1/3}}{2}, -\frac{(-4-4I\sqrt{3})^{1/3}}{4} - \frac{I\sqrt{3}(-4-4I\sqrt{3})^{1/3}}{4}, -\frac{(-4-4I\sqrt{3})^{1/3}}{4} + \frac{I\sqrt{3}(-4-4I\sqrt{3})^{1/3}}{4}$$

$$1,\ -1,\ \sqrt{\frac{\sqrt{5}}{4}-\frac{1}{4}+\frac{I\sqrt{2}\sqrt{5+\sqrt{5}}}{4}},\ \sqrt{-\frac{\sqrt{5}}{4}-\frac{1}{4}+\frac{I\sqrt{2}\sqrt{5-\sqrt{5}}}{4}},$$
$$\sqrt{-\frac{\sqrt{5}}{4}-\frac{1}{4}-\frac{I\sqrt{2}\sqrt{5-\sqrt{5}}}{4}},\ \sqrt{\frac{\sqrt{5}}{4}-\frac{1}{4}-\frac{I\sqrt{2}\sqrt{5+\sqrt{5}}}{4}},$$
$$-\sqrt{\frac{\sqrt{5}}{4}-\frac{1}{4}+\frac{I\sqrt{2}\sqrt{5+\sqrt{5}}}{4}},\ -\sqrt{-\frac{\sqrt{5}}{4}-\frac{1}{4}+\frac{I\sqrt{2}\sqrt{5-\sqrt{5}}}{4}},$$
$$-\sqrt{-\frac{\sqrt{5}}{4}-\frac{1}{4}-\frac{I\sqrt{2}\sqrt{5-\sqrt{5}}}{4}},\ -\sqrt{\frac{\sqrt{5}}{4}-\frac{1}{4}-\frac{I\sqrt{2}\sqrt{5+\sqrt{5}}}{4}}$$
$$1,\ \cos\left(\frac{2\pi}{11}\right)+I\sin\left(\frac{2\pi}{11}\right),\ \cos\left(\frac{4\pi}{11}\right)+I\sin\left(\frac{4\pi}{11}\right),\ -\cos\left(\frac{5\pi}{11}\right)+I\sin\left(\frac{5\pi}{11}\right),$$
$$-\cos\left(\frac{3\pi}{11}\right)+I\sin\left(\frac{3\pi}{11}\right),\ -\cos\left(\frac{\pi}{11}\right)+I\sin\left(\frac{\pi}{11}\right),\ -\cos\left(\frac{\pi}{11}\right)$$
$$-I\sin\left(\frac{\pi}{11}\right),\ -\cos\left(\frac{3\pi}{11}\right)-I\sin\left(\frac{3\pi}{11}\right),\ -\cos\left(\frac{5\pi}{11}\right)-I\sin\left(\frac{5\pi}{11}\right),$$
$$\cos\left(\frac{4\pi}{11}\right)-I\sin\left(\frac{4\pi}{11}\right),\ \cos\left(\frac{2\pi}{11}\right)-I\sin\left(\frac{2\pi}{11}\right)$$
$$1,\ -1,\ I,\ -I,\ \frac{\sqrt{2}\left(-2-2I\sqrt{3}\right)^{1/4}}{2},\ -\frac{\sqrt{2}\left(-2-2I\sqrt{3}\right)^{1/4}}{2}, \qquad \mathbf{(1.3)}$$
$$\frac{\sqrt{2}\left(-2+2I\sqrt{3}\right)^{1/4}}{2},\ -\frac{\sqrt{2}\left(-2+2I\sqrt{3}\right)^{1/4}}{2},\ \frac{\sqrt{-2\sqrt{-2-2I\sqrt{3}}}}{2},$$
$$-\frac{\sqrt{-2\sqrt{-2-2I\sqrt{3}}}}{2},\ \frac{\sqrt{-2\sqrt{-2+2I\sqrt{3}}}}{2},\ -\frac{\sqrt{-2\sqrt{-2+2I\sqrt{3}}}}{2}$$

Aufgabe 22

```
> restart:
```

(a)

```
> Gl := 2^x - 2*sqrt(x) = 0;
```

$$Gl := 2^x - 2\sqrt{x} = 0 \qquad \mathbf{(2.1)}$$

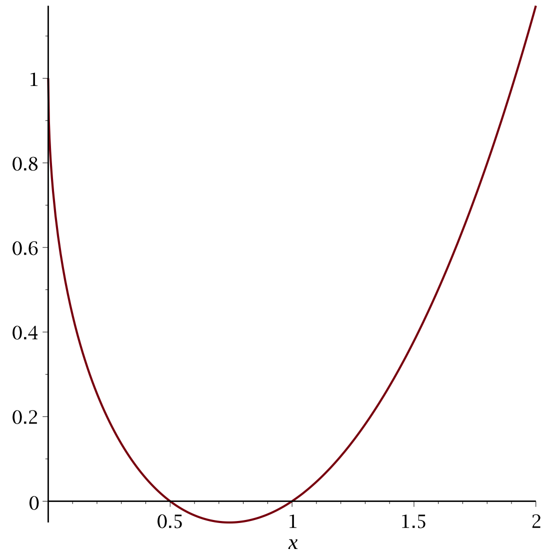
```
> solve(Gl, x);
```

$$\frac{1}{2},\ 1 \qquad \mathbf{(2.2)}$$

```
> # Test
> subs(x = 1/2, lhs(Gl));
> subs(x = 1, lhs(Gl));
> plot(lhs(Gl), x = 0..2);
```

$$0$$

$$0$$



(b)

```
> Gl := 4^x - 4 * x^(3/4) = 0;
```

$$Gl := 4^x - 4\,x^{3/4} = 0 \tag{2.3}$$

```
> sols := solve(Gl, x);
> allvalues([ sols ]);
> #hmm
```

$$sols := \frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ 1.122462048\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ 1.587401052\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ -2.514006884 - 0.8446145512\,I\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ -2.514006884+0.8446145512\,I\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ -1.035480887-2.857174478\,I\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ -0.8911608334-2.594476763\,I\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ -0.6518335788-2.205612846\,I\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ -0.5186642569-0.8082944879\,I\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ -0.5186642569+0.8082944879\,I\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 0.4025384095-1.873678593\,I\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 0.4025384095+1.873678593\,I\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 2.323341529-0.7285593618\,I\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 2.323341529+0.7285593618\,I\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 2.667872933-0.9325695489\,I\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 2.667872933+0.9325695489\,I\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 2.912489134-1.064643870\,I\right)^4}{8},$$

$$\frac{2^{1/3}\, RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ 2.912489134 + 1.064643870\, \mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\, RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ 3.106737308 - 1.164395090\, \mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\, RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ 3.106737308 + 1.164395090\, \mathrm{I}\right)^4}{8}$$

$$\left[\frac{2^{1/3}\, RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ 1.122462048\right)^4}{8}, \right. \qquad \textbf{(2.4)}$$

$$\frac{2^{1/3}\, RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ 1.587401052\right)^4}{8},$$

$$\frac{2^{1/3}\, RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ -2.514006884 - 0.8446145512\, \mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\, RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ -2.514006884 + 0.8446145512\, \mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\, RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ -1.035480887 - 2.857174478\, \mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\, RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ -0.8911608334 - 2.594476763\, \mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\, RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ -0.6518335788 - 2.205612846\, \mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\, RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ -0.5186642569 - 0.8082944879\, \mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\, RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ -0.5186642569 + 0.8082944879\, \mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\, RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}} + \_Z^3,\ 0.4025384095 - 1.873678593\, \mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 0.4025384095+1.873678593\,\mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 2.323341529-0.7285593618\,\mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 2.323341529+0.7285593618\,\mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 2.667872933-0.9325695489\,\mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 2.667872933+0.9325695489\,\mathrm{I}\right)^4}{8},$$

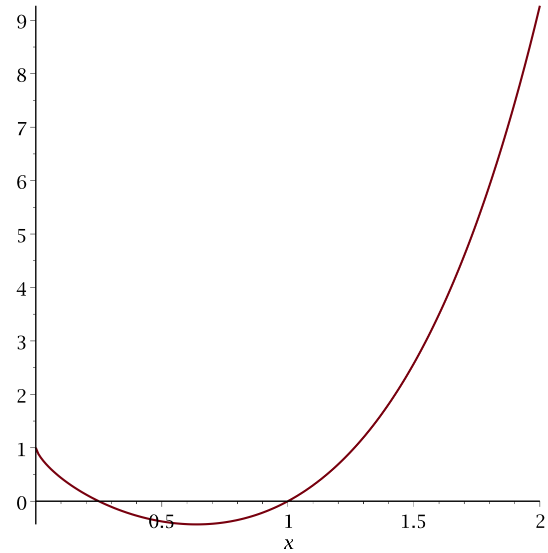
$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 2.912489134-1.064643870\,\mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 2.912489134+1.064643870\,\mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 3.106737308-1.164395090\,\mathrm{I}\right)^4}{8},$$

$$\frac{2^{1/3}\,RootOf\left(-2^{\frac{2^{1/3}\_Z^4}{4}}+\_Z^3,\ 3.106737308+1.164395090\,\mathrm{I}\right)^4}{8}\Bigg]$$

```
> plot(lhs(Gl), x = 0..2);
```




```
> evalf(sols);
```

0.2499999998, 1.000000000, 2.110698140 + 7.499940672 I, 2.110698140 − 7.499940672 I, 2.405412074 − 13.21640904 I, 2.183837316 − 8.647621088 I, 1.802372049 − 4.021117526 I, −0.08745730156 − 0.1015010617 I, −0.08745730156 + 0.1015010617 I, 1.407633339 + 1.591048720 I, 1.407633339 − 1.591048720 I, 1.925789514 − 5.189949494 I, 1.925789514 + 5.189949494 I, 2.248271710 − 9.792441790 I, 2.248271710 + 9.792441790 I, 2.449084378 − 14.35548019 I, 2.449084378 + 14.35548019 I, 2.595324236 − 18.90544998 I, 2.595324236 + 18.90544998 I **(2.5)**

(c)

```
> # Finde reele Lösungen
> map(sol -> is(evalf(sol), real), [ sols ]);
> solsReal := [ sols[1], sols[2] ];
> solsRealNumerical := map(x -> evalf(x), solsReal) ;
```

[*true*, *true*, *false*, *false*, *false*, *false*, *false*, *false*, *false*, *false*, *false*, *false*, *false*, *false*, *false*, *false*, *false*, *false*, *false*]

$$solsReal := \left[\frac{2^{1/3}\, RootOf\left(-2\,\frac{2^{1/3}\_Z^4}{4} + \_Z^3,\ 1.122462048\right)^4}{8}, \frac{2^{1/3}\, RootOf\left(-2\,\frac{2^{1/3}\_Z^4}{4} + \_Z^3,\ 1.587401052\right)^4}{8} \right]$$

$$solsRealNumerical := [0.2499999998,\ 1.000000000] \quad \textbf{(2.6)}$$

```
> # oder kürzer (aber nicht in der VL)
> solsRealNumerical := map(x -> evalf(x), select(sol -> is(evalf(sol), real), [ sols ]));
```

$$solsRealNumerical := [0.2499999998,\ 1.000000000] \quad \textbf{(2.7)}$$

```
> # Teste analytische Nullstellen
> subs(x = 1/4, lhs(Gl));
```

$$0 \quad \textbf{(2.8)}$$

```
> subs(x = 1, lhs(Gl));
```

$$0 \quad \textbf{(2.9)}$$

Aufgabe 23

```
> restart:
```

(a)

```
> simplify(sin(4*x) * cos(2*x));
```

$$2\sin(2x)\cos(2x)^2 \quad \textbf{(3.1)}$$

```
> simplify((1/2) * sin(6*x) + (1/2) * sin(2*x));
```

$$2\sin(2x)\cos(2x)^2 \quad \textbf{(3.2)}$$

(b)

```
> f := x -> cos(4*x);
```

$$f := x \mapsto \cos(4x) \quad \textbf{(3.3)}$$

```
> expand(f(x));
```

$$8\cos(x)^4 - 8\cos(x)^2 + 1 \quad \textbf{(3.4)}$$

```
> ?trigsubs
> gs := trigsubs(f(x));
```

$$gs := \left[\cos(-4x),\ \cos(2x)^2 - \sin(2x)^2,\ \frac{1}{\sec(4x)},\ \frac{1-\tan(2x)^2}{1+\tan(2x)^2},\ \frac{e^{4Ix}}{2} + \frac{e^{-4Ix}}{2},\ 2\cos(3x)\cos(x) - \cos(2x),\ \sin(x)^4 - 6\cos(x)^2\sin(x)^2 + \cos(x)^4\right] \quad \textbf{(3.5)}$$

```
> map(g -> simplify(expand(g)), gs);
```

$$[8\cos(x)^4 - 8\cos(x)^2 + 1,\ 8\cos(x)^4 - 8\cos(x)^2 + 1,\ 8\cos(x)^4 - 8\cos(x)^2 \quad \textbf{(3.6)}$$

$+1,\ 8\cos(x)^4-8\cos(x)^2+1,\ \cos(4x),\ 8\cos(x)^4-8\cos(x)^2+1,$
$8\cos(x)^4-8\cos(x)^2+1]$

(c)

```
> h := x -> sin(x) * sin(y) * sin(z);
```

$$h := x \mapsto \sin(x)\,\sin(y)\,\sin(z) \quad (3.7)$$

```
> combine(h(x));
```

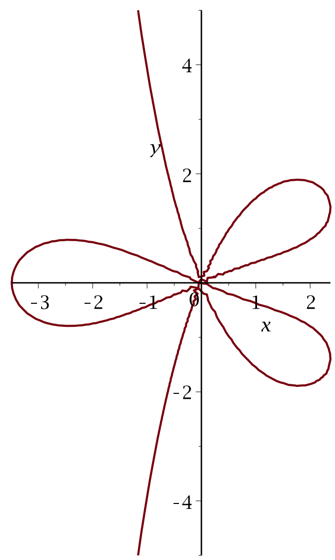
$$\frac{\sin(z+x-y)}{4}-\frac{\sin(-z+x-y)}{4}-\frac{\sin(z+x+y)}{4}+\frac{\sin(-z+x+y)}{4} \quad (3.8)$$

Aufgabe 24

```
> restart:
> with(plots):
> R := z -> (1 + 1/3 * z)/(1 - 2/3 * z + 1/6 * z^2);
```

$$R := z \mapsto \frac{1+\frac{z}{3}}{1-\frac{2}{3}\,z+\frac{1}{6}\,z^2} \quad (4.1)$$

```
> implicitplot(abs(exp(-x - I*y) * R(x + I * y)) = 1, x = -5..10,
  y = -5..5, numpoints = 10000, scaling = constrained);
```



4
2
-2
-4
y
-3
-2
-1
0
1
2
x