

Blatt 5

Aufgabe 17

```
> restart:
```

```
(a)
```

```
> f := x -> cos(5 * arccos(x)) + x: 'f'(x) = f(x);
```

```
> df := x -> diff(f(x), x): 'df'(x) = df(x);
```

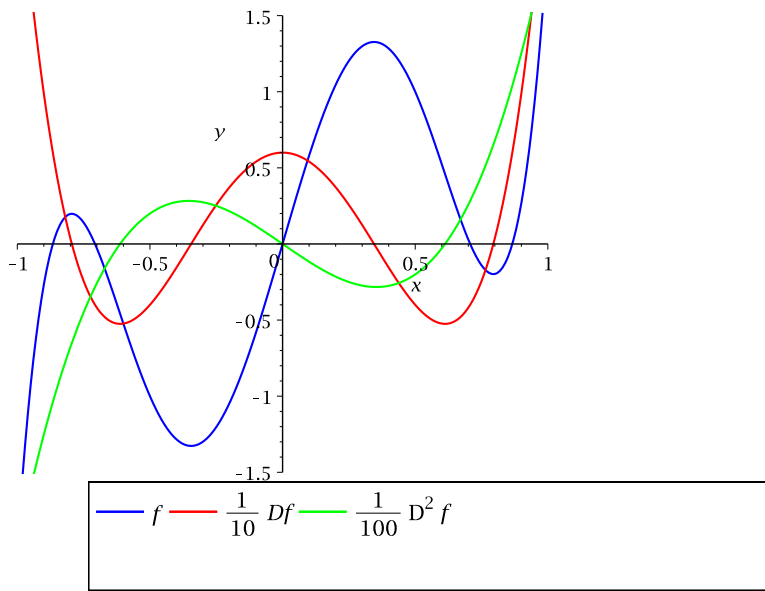
```
> d2f := x -> diff(f(x), x$2): 'd2f'(x) = d2f(x);
```

$$f(x) = \cos(5 \arccos(x)) + x$$

$$df(x) = \frac{5 \sin(5 \arccos(x))}{\sqrt{-x^2 + 1}} + 1$$

$$d2f(x) = -\frac{25 \cos(5 \arccos(x))}{-x^2 + 1} + \frac{5 \sin(5 \arccos(x)) x}{(-x^2 + 1)^{3/2}} \quad (1.1)$$

```
> plot([ f(x), df(x) / 10, d2f(x) / 100 ], x = -1..1, y = -1.5.  
.1.5, color = [ blue, red, green ], legend = [ 'f', 'Df/10',  
'D^2*f/100' ]);
```



(b)

> $g := x \rightarrow (x^4 - 7 * x^2 + 3) / (x^4 + 5): 'g'(x) = g(x);$

$$g(x) = \frac{x^4 - 7x^2 + 3}{x^4 + 5} \quad (1.2)$$

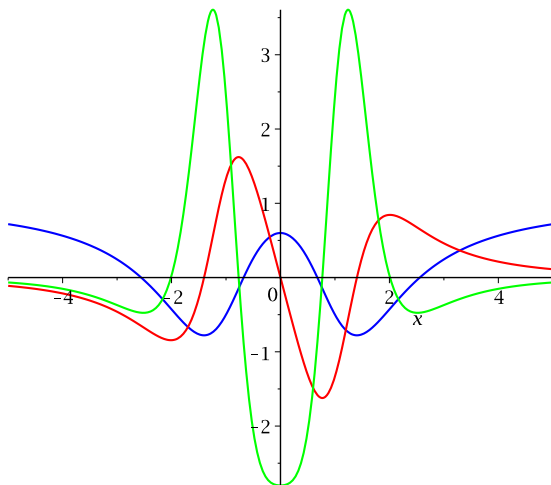
> $dg := \text{diff}(g(x), x): 'dg'(x) = dg(x);$

$$dg(x) = \frac{4x(x)^3 - 14x(x)}{x(x)^4 + 5} - \frac{4(x(x)^4 - 7x(x)^2 + 3)x(x)^3}{(x(x)^4 + 5)^2} \quad (1.3)$$

> $d2g := \text{diff}(g(x), x\$2): 'd2g'(x) = d2g(x);$

$$d2g(x) = \frac{12x(x)^2 - 14}{x(x)^4 + 5} - \frac{8(4x(x)^3 - 14x(x))x(x)^3}{(x(x)^4 + 5)^2} + \frac{32(x(x)^4 - 7x(x)^2 + 3)x(x)^6}{(x(x)^4 + 5)^3} - \frac{12(x(x)^4 - 7x(x)^2 + 3)x(x)^2}{(x(x)^4 + 5)^2} \quad (1.4)$$

> $\text{plot}([g(x), dg(x), d2g(x)], x = -5..5, \text{color} = [\text{blue}, \text{red}, \text{green}], \text{legend} = ['g', 'Dg/10', 'D^2*g/100']);$



— g
 — $\frac{1}{10} Dg$
 — $\frac{1}{100} D^2 g$

▼ Aufgabe 18

```

> restart:
> print("Zahl", "Gerade", "Prim", "Natürlich", "Durch 3 Teilbar")
;
> for number in seq(kk, kk = -10..10) do
  print(number, is(number, even), is(number, natural), is
(number, prime), is(number/3, integer));
end do;
  "Zahl", "Gerade", "Prim", "Natürlich", "Durch 3 Teilbar"
    -10, true, false, false, false
    -9, false, false, false, true
    -8, true, false, false, false
    -7, false, false, false, false
    -6, true, false, false, true
    -5, false, false, false, false
    -4, true, false, false, false

```

```
-3, false, false, false, true
-2, true, false, false, false
-1, false, false, false, false
0, true, false, false, true
1, false, true, false, false
2, true, true, true, false
3, false, true, true, true
4, true, true, false, false
5, false, true, true, false
6, true, true, false, true
7, false, true, true, false
8, true, true, false, false
9, false, true, false, true
10, true, true, false, false
```

(2.1)

Aufgabe 19

```
> restart:
```

```
(a)
```

```
> ggt := proc (m::integer, n::integer)::integer;
  description "Berechnet den größten gemeinsamen Teiler zweier
natürlicher Zahlen";
  local j := m, k := n, l := 1;
  if not is(m, natural) or not is(n, natural) then
    error("Erlaube für m und n nur natürliche Zahlen.");
  end if;
  while not l = 0 do
    l := j mod k;
    #print('j' = j, 'k' = k, 'j mod k' = l);
    j := k;
    k := l;
  end do;
  # Die letzte Ausgabe ist die Ausgabe der gesamten Funktion
  j;
end proc:
> # # Test
> # 'ggt(2, 3)' = ggt(2, 3);
> # ggt(4, 2);
> # ggt(2, 4);
> # ggt(1, 2);
> # ggt(2, 1);
```

```
(b)
```

```
> ggt(19278, 3234);
```

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(3.1)

Aufgabe 20

```
> restart;
> ?rem
> ?degree
> ggTpoly := proc(p::polynom, q::polynom)::polynom;
  description "Berechnet den größten gemeinsamen Teiler zweier
  Polynome";
  local r := p, s := q, t := x;
  while degree(t, x) >= 0 do
    t := rem(r, s, x);
    #print(r, s, t);
    r := s;
    s := t;
  end do;
  # Ausgabe, normalisieren (Führkoeffizient 1)
  r := r / lcoeff(r);
end proc;
> # # Test
> # simplify(ggTpoly(x^3 - 1, x - I^(4/3)) - gcd(x^3 - 1, x - I^
(4/3)));
> # simplify(ggTpoly(x^3 - 1, x - I^(4/3)) - ggTpoly(x - I^(4/3),
x^3 - 1));
```

(b)

```
> p := x^4 + x^3 - x^2 + x + 2;
> q := x^3 + 2*x^2 + 2*x + 1;
> 'ggT'(p, q) = ggTpoly(p, q);
       $ggT(x^4 + x^3 - x^2 + x + 2, x^3 + 2x^2 + 2x + 1) = x + 1$  (4.1)
```

```
> p := x^4 + x^3 + x + 1;
> q := x^2 - 1;
> 'ggT'(p, q) = ggTpoly(p, q);
       $ggT(x^4 + x^3 + x + 1, x^2 - 1) = x + 1$  (4.2)
```