# Blatt 4

## ▼ Aufgabe 13

```
> restart:
> A := [ 1, Pi, exp(1), pi, x+y ];
```
$$A := [1, \pi, e, \pi, x + y] \tag{1.1}$$

```
> op(A);
```
$$1, \pi, e, \pi, x + y \tag{1.2}$$

```
> 'whattype'('op'(1, 'A')) = whattype(op(1, A));
> 'whattype'('op'(2, 'A')) = whattype(op(2, A));
> 'whattype'('op'(3, 'A')) = whattype(op(3, A));
> 'whattype'('op'(4, 'A')) = whattype(op(4, A));
> 'whattype'('op'(5, 'A')) = whattype(op(5, A));
```
$$whattype(op(1, A)) = integer$$
$$whattype(op(2, A)) = symbol$$
$$whattype(op(3, A)) = function$$
$$whattype(op(4, A)) = symbol$$
$$whattype(op(5, A)) = {}^{`}+{}^{`} \tag{1.3}$$

```
> op(op(3, A));
> 'whattype'('op'('op'(3, 'A'))) = whattype(op(op(3, A)));
```
$$1$$
$$whattype(op(op(3, A))) = integer \tag{1.4}$$

```
> op(op(5, A));
> 'whattype'('op'(1, 'op'(5, 'A'))) = whattype(op(1, op(5, A)));
> 'whattype'('op'(2, 'op'(5, 'A'))) = whattype(op(2, op(5, A)));
```
$$x, y$$
$$whattype(op(1, op(5, A))) = symbol$$
$$whattype(op(2, op(5, A))) = symbol \tag{1.5}$$

```
> B := { x * y / z, u / v * w, (alpha + beta) / (beta * delta) };
```
$$B := \left\{ \frac{u\,w}{v}, \frac{x\,y}{z}, \frac{\alpha + \beta}{\beta\,\delta} \right\} \tag{1.6}$$

```
> op(B);
```
$$\frac{u\,w}{v}, \frac{x\,y}{z}, \frac{\alpha + \beta}{\beta\,\delta} \tag{1.7}$$

```
> whattype(op(1, B));
> whattype(op(2, B));
> whattype(op(3, B));
```
$${}^{`}*{}^{`}$$
$${}^{`}*{}^{`}$$
$${}^{`}*{}^{`} \tag{1.8}$$

```
> op(op(1, B));
```

$$u, \frac{1}{v}, w \qquad\qquad (1.9)$$

```
> whattype(op(1, op(1, B)));
> whattype(op(2, op(1, B)));
> whattype(op(3, op(1, B)));
```

$$symbol$$
$$`\wedge`$$
$$symbol \qquad\qquad (1.10)$$

```
> op(op(2, op(1, B)));
```

$$v, -1 \qquad\qquad (1.11)$$

```
> whattype(op(1, op(2, op(1, B))));
> whattype(op(2, op(2, op(1, B))));
```

$$symbol$$
$$integer \qquad\qquad (1.12)$$

```
> op(op(2, B));
```

$$x, y, \frac{1}{z} \qquad\qquad (1.13)$$

```
> whattype(op(1, op(2, B)));
> whattype(op(2, op(2, B)));
> whattype(op(3, op(2, B)));
```

$$symbol$$
$$symbol$$
$$`\wedge` \qquad\qquad (1.14)$$

```
> op(op(3, op(2, B)));
```

$$z, -1 \qquad\qquad (1.15)$$

```
> whattype(op(1, op(3, op(2, B))));
> whattype(op(2, op(3, op(2, B))));
```

$$symbol$$
$$integer \qquad\qquad (1.16)$$

```
> op(op(3, B));
```

$$\alpha + \beta, \frac{1}{\beta}, \frac{1}{\delta} \qquad\qquad (1.17)$$

```
> whattype(op(1, op(3, B)));
> whattype(op(2, op(3, B)));
> whattype(op(3, op(3, B)));
```

$$`C`$$
$$`\wedge`$$
$$`\wedge` \qquad\qquad (1.18)$$

```
> op(op(1, op(3, B)));
```

$$\alpha, \beta \qquad\qquad (1.19)$$

```
> whattype(op(1, op(1, op(3, B))));
> whattype(op(2, op(1, op(3, B))));
```
$$symbol$$

$$symbol \tag{1.20}$$

```
> op(op(2, op(3, B)));
```
$$\beta, -1 \tag{1.21}$$

```
> whattype(op(1, op(2, op(3, B))));
> whattype(op(2, op(2, op(3, B))));
```
$$symbol$$

$$integer \tag{1.22}$$

```
> op(op(3, op(3, B)));
```
$$\delta, -1 \tag{1.23}$$

```
> whattype(op(1, op(3, op(3, B))));
> whattype(op(2, op(3, op(3, B))));
```
$$symbol$$

$$integer \tag{1.24}$$

# ▼ Aufgabe 14

```
> restart:
> f := x -> x^5 - 5*x^4 - 10*x^3 + 50*x^2 + 9*x - 45;
```
$$f := x \mapsto x^5 - 5\,x^4 - 10\,x^3 + 50\,x^2 + 9\,x - 45 \tag{2.1}$$

```
> factor(f); # Test: Nullstellen bei -3, -1, 1, 3, 5
```
$$f \tag{2.2}$$

```
> L1 := [ seq(i - 5, i = 0..7) ];
```
$$L1 := [-5, -4, -3, -2, -1, 0, 1, 2] \tag{2.3}$$

```
> L1 := [ seq(i, i = -5..2) ]; # Wahlweise
```
$$L1 := [-5, -4, -3, -2, -1, 0, 1, 2] \tag{2.4}$$

```
> L1 := [ -5, -4, -3, -2, -1, 0, 1, 2 ]; # Wahlweise
```
$$L1 := [-5, -4, -3, -2, -1, 0, 1, 2] \tag{2.5}$$

```
> Liste1 := map(f, L1);
```
$$Liste1 := [-3840, -945, 0, 105, 0, -45, 0, 45] \tag{2.6}$$

```
> L2 := [ seq(i - 1, i = 0..8) ];
```
$$L2 := [-1, 0, 1, 2, 3, 4, 5, 6, 7] \tag{2.7}$$

```
> L2 := [ seq(i, i = -1..7) ]; # Wahlweise
```
$$L2 := [-1, 0, 1, 2, 3, 4, 5, 6, 7] \tag{2.8}$$

```
> L2 := [ -1, 0, 1, 2, 3, 4, 5, 6, 7 ]; # Wahlweise
```
$$L2 := [-1, 0, 1, 2, 3, 4, 5, 6, 7] \tag{2.9}$$

```
> Liste2 := map(f, L2);
```
$$Liste2 := [0, -45, 0, 45, 0, -105, 0, 945, 3840] \tag{2.10}$$

```
> A := convert(Liste1, set);
```
$$A := \{-3840, -945, -45, 0, 45, 105\}$$ **(2.11)**

```
> B := convert(Liste2, set);
```
$$B := \{-105, -45, 0, 45, 945, 3840\}$$ **(2.12)**

```
> ?union
> C := A union B: 'A' union 'B' = C;
```
$$A \cup B = \{-3840, -945, -105, -45, 0, 45, 105, 945, 3840\}$$ **(2.13)**

```
> nops(C);
```
$$9$$ **(2.14)**

## ▼ Aufgabe 15

```
> restart:
```
(a)
```
> P[0] := 1;
> P[1] := x;
> for n from 1 to 9 do
    P[n+1] := factor(1/(n+1) * ((2*n+1)*x*P[n] - n*P[n-1]));
  od;
```

$$P_0 := 1$$

$$P_1 := x$$

$$P_2 := \frac{3\,x^2}{2} - \frac{1}{2}$$

$$P_3 := \frac{x\,(5\,x^2 - 3)}{2}$$

$$P_4 := \frac{35}{8}\,x^4 - \frac{15}{4}\,x^2 + \frac{3}{8}$$

$$P_5 := \frac{x\,(63\,x^4 - 70\,x^2 + 15)}{8}$$

$$P_6 := \frac{231}{16}\,x^6 - \frac{315}{16}\,x^4 + \frac{105}{16}\,x^2 - \frac{5}{16}$$

$$P_7 := \frac{x\,(429\,x^6 - 693\,x^4 + 315\,x^2 - 35)}{16}$$

$$P_8 := \frac{6435}{128}\,x^8 - \frac{3003}{32}\,x^6 + \frac{3465}{64}\,x^4 - \frac{315}{32}\,x^2 + \frac{35}{128}$$

$$P_9 := \frac{x\,(12155\,x^8 - 25740\,x^6 + 18018\,x^4 - 4620\,x^2 + 315)}{128}$$

$$P_{10} := \frac{46189}{256}\,x^{10} - \frac{109395}{256}\,x^8 + \frac{45045}{128}\,x^6 - \frac{15015}{128}\,x^4 + \frac{3465}{256}\,x^2 - \frac{63}{256}$$ **(3.1)**

(b)
```
> Integral := (m, n) -> int(P[m]*P[n], x = -1..1);
```

$$Integral := (m, n) \mapsto \int_{-1}^{1} P_m P_n \, dx \qquad\qquad \textbf{(3.2)}$$

```
> # Results := seq(seq(Integral(m, n), m = 0..10), n = 0..10);
> allOkay := true:
> for m from 0 to 10 do
    for n from 0 to 10 do
      if (m = n) then
        answer := is(Integral(m, n) = 2 / (2*n + 1));
       else
        answer := is(Integral(m, n) = 0);
       fi;
     allOkay := allOkay and answer;
    end do;
  end do:
> if allOkay then
    print("Formel korrekt.");
  else
    error("Formel nicht korrekt.");
  end if;
```
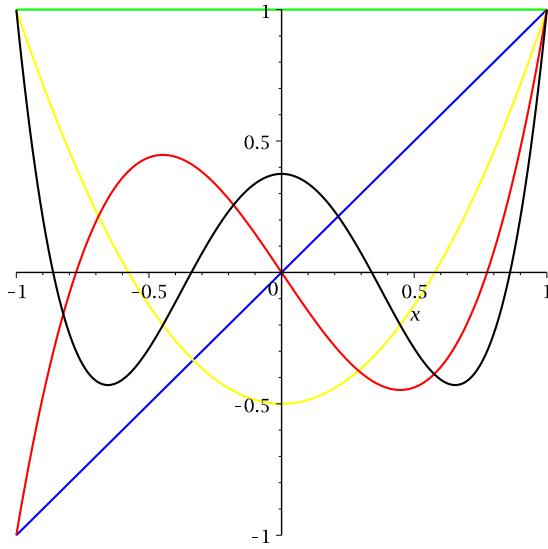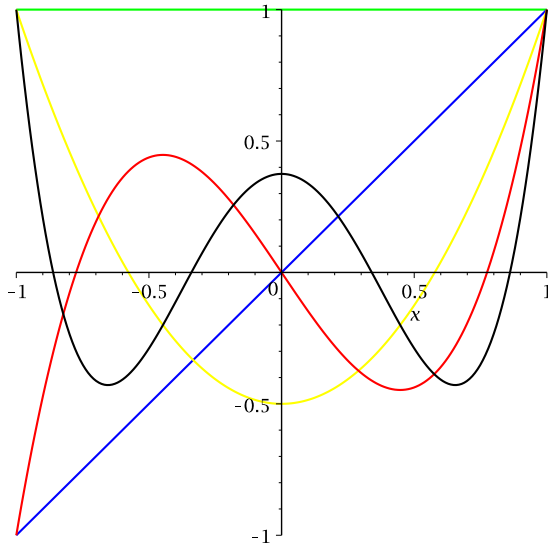
$$\text{"Formel korrekt."} \qquad\qquad \textbf{(3.3)}$$

(c)

```
> with(plots):
> colors := [ 'green', 'blue', 'yellow', 'red', 'black' ];
```

$$colors := [green, \; blue, \; yellow, \; red, \; black] \qquad\qquad \textbf{(3.4)}$$

```
> p = []:
> for n from 0 to 4 do
    p[n] := plot(P[n], x = -1..1, color = colors[n + 1]);
  end do:
> display([ seq(p[n], n = 0..4) ]);
```
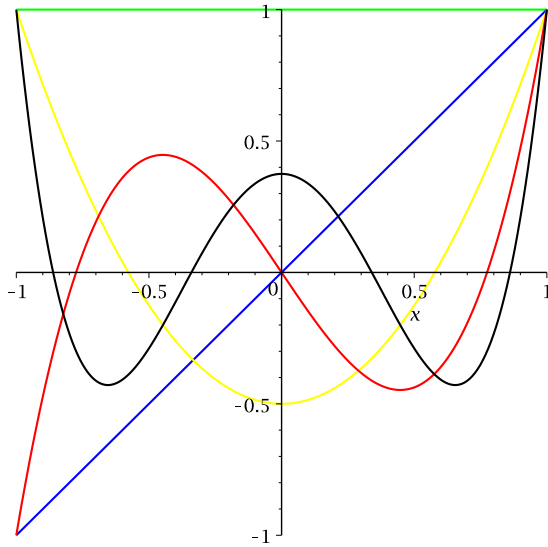
```
> plot([ P[0], P[1], P[2], P[3], P[4] ], x = -1..1, color = [
  'green', 'blue', 'yellow', 'red', 'black' ]);
```

# Aufgabe 16

```
> restart:
```
(a)
```
> # Ausgabe wird durch ':' unterdrückt
> f[0] := 1:
> f[1] := 1:
> for kk from 2 to 100 do
    f[kk] := f[kk-1] + f[kk-2];
  od:
> # # Ausgabe, wenn man möchte
> # [ seq(f[kk], kk = 0..100) ];
```
(b)
```
> ?ithprime
> Start := 9; Ende := 25;
```

$$Start := 9$$
$$Ende := 25$$

**(4.1)**

```
> primes := [ seq(ithprime(kk), kk = Start..Ende) ];
```
$$primes := [23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97]$$  **(4.2)**

```
> # Anzahl testen
> nops(primes) = Ende - Start + 1;
```
$$17 = 17$$  **(4.3)**

```
> for kk in primes do
    f[kk];
  od;
```
$$46368$$
$$832040$$
$$2178309$$
$$39088169$$
$$267914296$$
$$701408733$$
$$4807526976$$
$$86267571272$$
$$1548008755920$$
$$4052739537881$$
$$72723460248141$$  **(4.4)**