Computergestuetzte Mathematik zur Analysis

Lektion 3 (Integration und Differentiation)

Summen und Reihen (Wdh.)

```
> restart:
```

```
> sum(j,j=0..n);
```

$$\frac{1}{2}\,(n+1)^2-\frac{1}{2}\,n-\frac{1}{2} \tag{1.1}$$

```
> sum(1/j^4, j = 1 .. infinity);
```

$$\frac{1}{90}\,\pi^4 \tag{1.2}$$

Grenzwerte / Limes

```
> limit((1+z/n)^n,n=infinity);
```

$$e^z \tag{2.1}$$

```
> limit( (x^2+2*x-3)/(x-1),x=1);
```

$$4 \tag{2.2}$$

```
> sum(1/j,j=1..n)-ln(n);
```

$$\Psi(n+1)+\gamma-\ln(n) \tag{2.3}$$

```
> limit( sum(1/j,j=1..n)-ln(n),n=infinity);
```

$$\gamma \tag{2.4}$$

Integration und Differentiation

```
> f:= 1/(1+t^2);
```

$$f:=\frac{1}{t^2+1} \tag{3.1}$$

```
> If := int(f,t);
```

$$If:=\arctan(t) \tag{3.2}$$

```
> diff(If,t);
```

$$\frac{1}{t^2+1} \tag{3.3}$$

```
> int(f,t=a..b);
```

$$-\arctan(a)+\arctan(b) \tag{3.4}$$

```
> ff := x -> 1/(1+x^2);
```

(3.5)

$$ff := x \to \frac{1}{x^2+1} \tag{3.5}$$

```
> int(ff,x); # Stimmt das?
```

$$ff\,x \tag{3.6}$$

```
> int(ff(x),x);
```

$$\arctan(x) \tag{3.7}$$

```
> int(ff(t),t);
```

$$\arctan(t) \tag{3.8}$$

```
> diff(ff,x);
```

$$0 \tag{3.9}$$

```
> diff(ff(y),y);
```

$$-\frac{2\,y}{(y^2+1)^2} \tag{3.10}$$

```
> restart;
> f := exp(Pi*I*x);
```

$$f := \mathrm{e}^{\mathrm{I}\pi x} \tag{3.11}$$

```
> diff(f,x);
```

$$\mathrm{I}\pi\,\mathrm{e}^{\mathrm{I}\pi x} \tag{3.12}$$

```
> diff(f,x,x);
```

$$-\pi^2\,\mathrm{e}^{\mathrm{I}\pi x} \tag{3.13}$$

```
> diff(f,x$4);
```

$$\pi^4\,\mathrm{e}^{\mathrm{I}\pi x} \tag{3.14}$$

```
> x$4;
```

$$x,\ x,\ x,\ x \tag{3.15}$$

Kettenregel:

```
> f:= 'f';
```

$$f := f \tag{3.16}$$

```
> g:= 'g';
```

$$g := g \tag{3.17}$$

```
> diff(f(g(x)),x);
```

$$\mathrm{D}(f)\,(g(x))\left(\frac{\mathrm{d}}{\mathrm{d}x}\,g(x)\right) \tag{3.18}$$

Produktregel:

```
> diff(f(x)*g(x),x);
```

$$\left(\frac{\mathrm{d}}{\mathrm{d}x}\,f(x)\right)g(x)+f(x)\left(\frac{\mathrm{d}}{\mathrm{d}x}\,g(x)\right) \tag{3.19}$$

Quotientenregel:

```
> # UA
```

Dito

Verweis auf das letzte,vorletzte und vorvorletzte berechnete Ergebnis

```
> a:= 2+alpha;
```

$$a := 2 + \alpha \tag{4.1}$$

```
> % + 2;
```

$$4 + \alpha \tag{4.2}$$

```
> b:=3+beta;
```

$$b := 3 + \beta \tag{4.3}$$

```
> %%%;
```

$$2 + \alpha \tag{4.4}$$

Traege Operatoren

```
> Limit(1/n,n=infinity);
```

$$\lim_{n \to \infty} \frac{1}{n} \tag{5.1}$$

```
> Sum(j,j=0..n);
```

$$\sum_{j=0}^{n} j \tag{5.2}$$

```
> value(%%);
```

$$0 \tag{5.3}$$

```
> value((5.2));
```

$$\frac{1}{2}\,(n+1)^2 - \frac{1}{2}\,n - \frac{1}{2} \tag{5.4}$$

```
> Sum(1/j^4, j = 1 .. infinity);
```

$$\sum_{j=1}^{\infty} \frac{1}{j^4} \tag{5.5}$$

```
> Limit((1+z/n)^n,n=infinity);
```

$$\lim_{n \to \infty} \left(1 + \frac{z}{n}\right)^n \tag{5.6}$$

```
> Limit( (x^2+2*x-3)/(x-1),x=1);
```

$$\lim_{x \to 1} \frac{x^2 + 2\,x - 3}{x - 1} \tag{5.7}$$

```
> S:=Sum(1/j,j=1..n)-ln(n);
```

$$S := \sum_{j=1}^{n} \frac{1}{j} - \ln(n) \tag{5.8}$$

```
> Limit(S,n=infinity);
```

$$\lim_{n\to\infty} \left(\sum_{j=1}^{n} \frac{1}{j} - \ln(n) \right) \tag{5.9}$$

```
> value((5.9));
```

$$\gamma \tag{5.10}$$

Differentiation II

```
> f := 1/(1+t^2);
  ff := t -> 1/(1+t^2);
```

$$f := \frac{1}{t^2+1}$$

$$ff := t \to \frac{1}{t^2+1} \tag{6.1}$$

```
> Diff(ff(t),t);
```

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{1}{t^2+1} \right) \tag{6.2}$$

```
> value(Diff(ff(t),t));
```

$$-\frac{2t}{(t^2+1)^2} \tag{6.3}$$

```
> diff(f,t);
```

$$-\frac{2t}{(t^2+1)^2} \tag{6.4}$$

```
> unapply(value(Diff(ff(t),t)),t);
  D(ff);
```

$$t \to -\frac{2t}{(t^2+1)^2}$$

$$t \to -\frac{2t}{(t^2+1)^2} \tag{6.5}$$

```
> D(arctan);  #Funktionsschreibweise
```

$$z \to \frac{1}{z^2+1} \tag{6.6}$$

```
> f:=exp@sin; #Verkettung
```

$$f := \exp@\sin \tag{6.7}$$

```
> f(x);
```

$$e^{\sin(x)} \tag{6.8}$$

```
> g:=exp@exp@exp;
```

$$g := \exp^{(3)} \tag{6.9}$$

```
> g(x);
```

$$\exp^{(3)}(x) \tag{6.10}$$

```
> expand(g(x));
```

$$e^{e^{e^x}} \tag{6.11}$$

```
> G:=exp@@3;
```

$$G := \exp^{(3)} \tag{6.12}$$

```
> G-g;
```

$$0 \tag{6.13}$$

```
> (D@@2)(f);
```

$$\exp@\sin\cos^2 - \exp@\sin\sin \tag{6.14}$$

```
> expand((6.14)(x));
```

$$e^{\sin(x)}\cos(x)^2 - e^{\sin(x)}\sin(x) \tag{6.15}$$

```
> f:='f'; g:='g';
```

$$f := f$$
$$g := g \tag{6.16}$$

```
> D(f@g);
```

$$D(f)@g\,D(g) \tag{6.17}$$

```
> expand((6.17)(x));
```

$$D(f)(g(x))\,D(g)(x) \tag{6.18}$$

```
> Quotientenregel % UA
Error, missing operator or `;`
```