Computergestuetzte Mathematik zur Analysis

Lektion 11 (14. Januar)

Gradienten und Vektorfelder

```
> restart:
> with(VectorCalculus):
> BasisFormat(false);
```

$$true \tag{1.1}$$

```
> f := a*x^2 + b*y^2 + c*z^2;
```

$$f := a\,x^2 + b\,y^2 + c\,z^2 \tag{1.2}$$

```
> gr := Gradient(f, [x, y, z]);
```

$$gr := \begin{bmatrix} 2\,a\,x \\ 2\,b\,y \\ 2\,c\,z \end{bmatrix} \tag{1.3}$$

```
> gr . <b*y, -a*x,0>;
```

$$0 \tag{1.4}$$

```
> vf := VectorField(<b*y, -a*x, 0>, cartesian[x,y,z]);
```

$$vf := \begin{bmatrix} b\,y \\ -a\,x \\ 0 \end{bmatrix} \tag{1.5}$$

```
> gr . vf; # Skalarprodukt
```

$$0 \tag{1.6}$$

Zeichnungen von Vektorfeldern

```
> restart:
> with(VectorCalculus):
> BasisFormat(false);
```

$$true \tag{2.1}$$

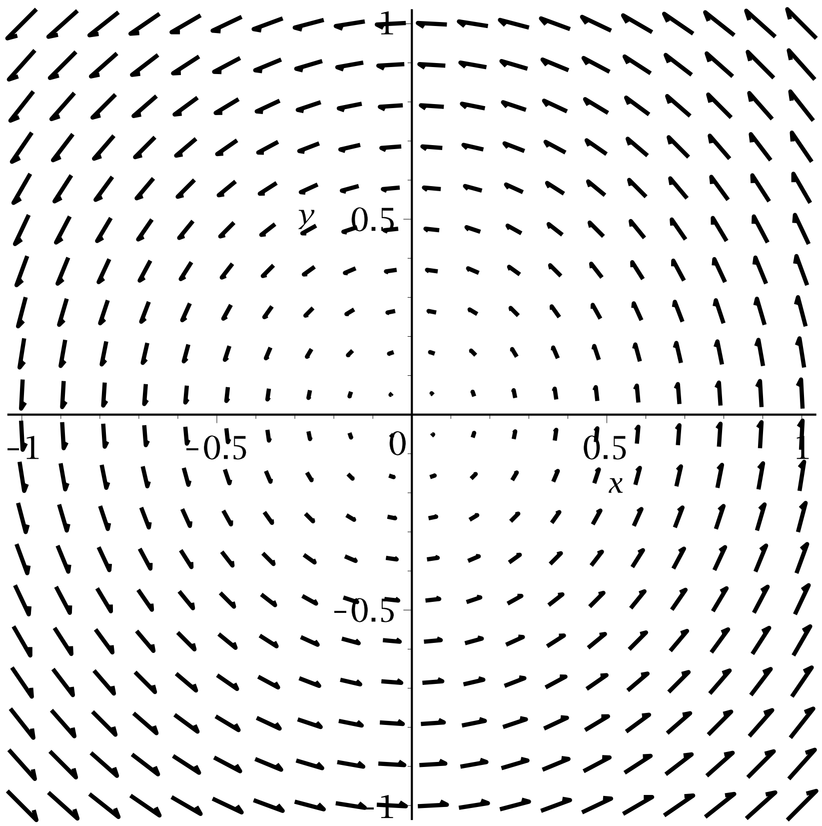
```
> vf1 := VectorField(<-y, x>, cartesian[x,y]);
  vf2 := VectorField(<x, y>, cartesian[x,y]);
  vf3 := VectorField(<y, x>, cartesian[x,y]);
```

$$vf1 := \begin{bmatrix} -y \\ x \end{bmatrix}$$

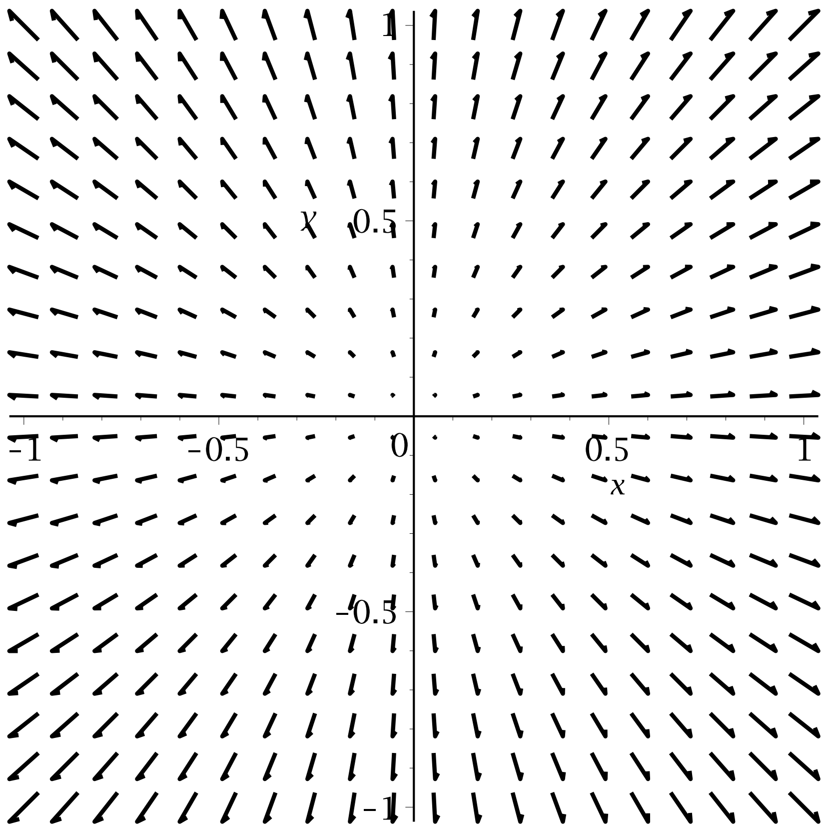
$$vf2 := \begin{bmatrix} x \\ y \end{bmatrix}$$

$$vf3 := \begin{bmatrix} y \\ x \end{bmatrix} \quad \textbf{(2.2)}$$

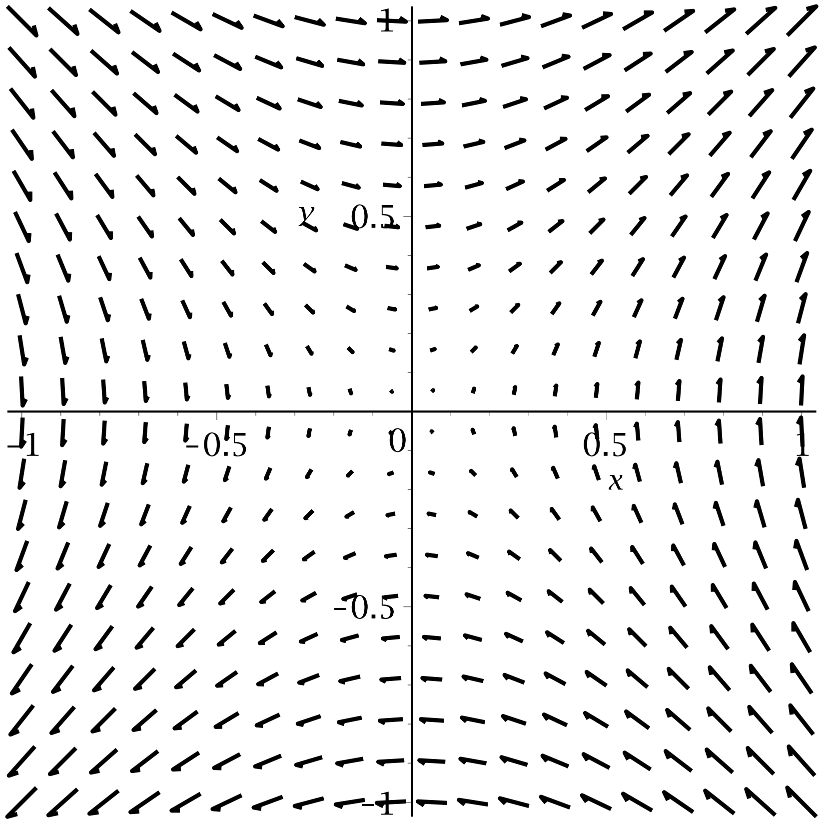
```
> with(plots):
> fieldplot(vf1, x = -1 .. 1, y = -1 .. 1, thickness = 2);
```




```
> fieldplot(vf2, x=-1..1,y=-1..1,thickness=2);
```



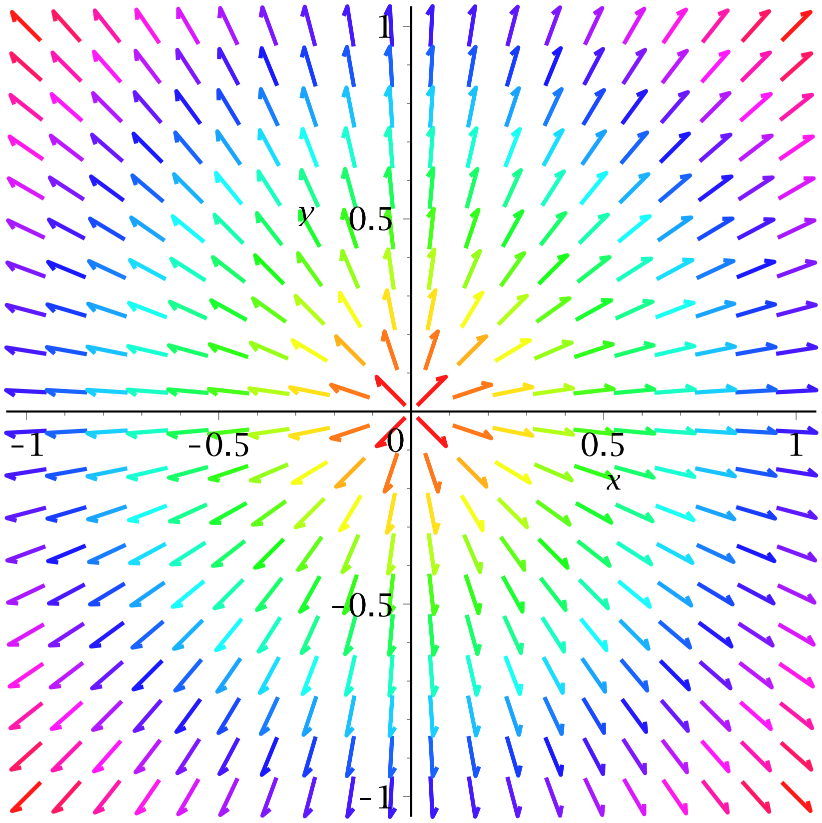

```
> fieldplot(vf3,x=-1..1,y=-1..1,thickness=2);
```




```
> with(LinearAlgebra):
> vf2 := vf2/Norm(vf2, 2);
```

$$vf2 := \begin{bmatrix} \dfrac{x}{\sqrt{|x|^2 + |y|^2}} \\ \dfrac{y}{\sqrt{|x|^2 + |y|^2}} \end{bmatrix} \tag{2.3}$$

```
> fieldplot(vf2, x = -1 .. 1, y = -1 .. 1, thickness = 2, color =
  sqrt(x^2 + y^2));
```



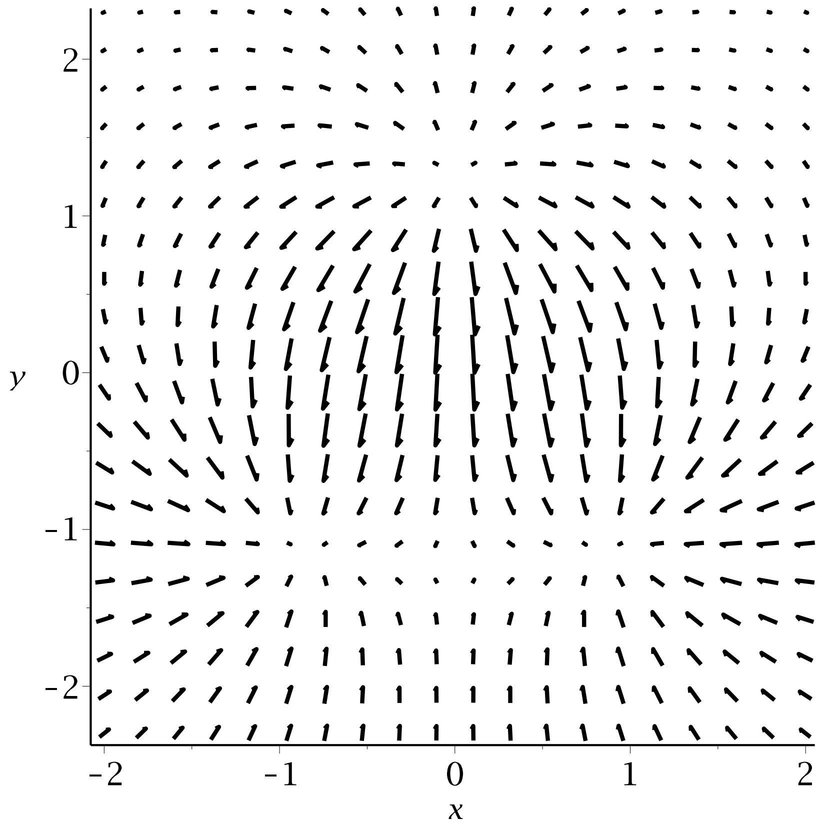

```
> k := -1/sqrt(x^2 + (y-1)^2 + 1) + 1/sqrt((x-1)^2 + (y+1)^2 + 1)
  + 1/sqrt((x+1)^2 + (y+1)^2 + 1);
```

$$k := -\frac{1}{\sqrt{x^2+(y-1)^2+1}} + \frac{1}{\sqrt{(x-1)^2+(y+1)^2+1}} + \frac{1}{\sqrt{(x+1)^2+(y+1)^2+1}} \quad \textbf{(2.4)}$$

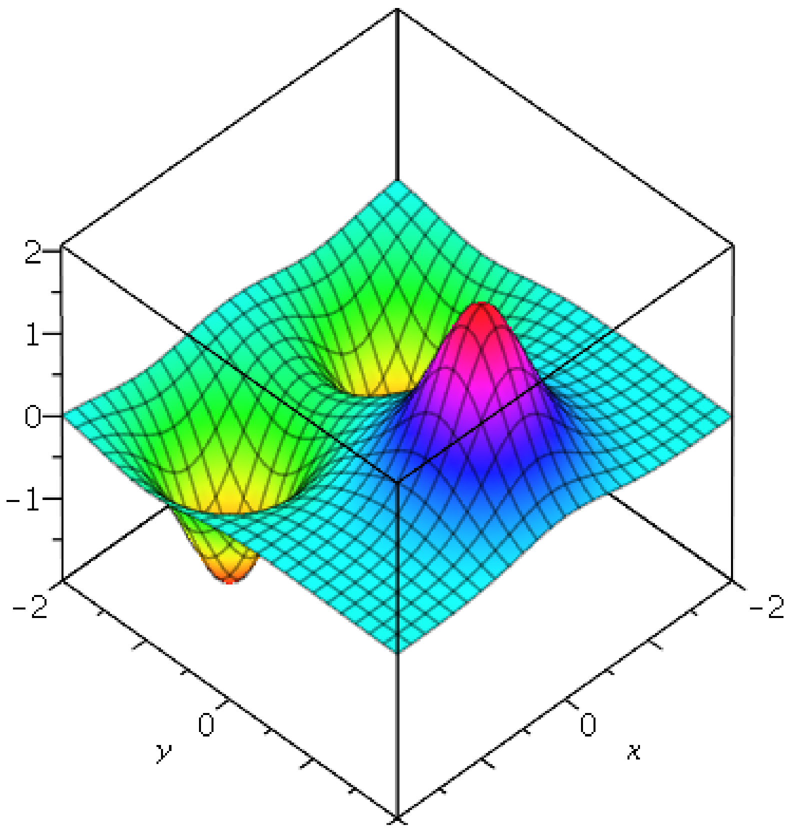
```
> gr := Gradient(k, [x,y]);
```

$$gr := \left[\left[\frac{x}{\left(x^2+(y-1)^2+1\right)^{3/2}} - \frac{1}{2}\,\frac{2\,x-2}{\left((x-1)^2+(y+1)^2+1\right)^{3/2}} - \frac{1}{2}\,\frac{2\,x+2}{\left((x+1)^2+(y+1)^2+1\right)^{3/2}}\right], \left[\frac{1}{2}\,\frac{2\,y-2}{\left(x^2+(y-1)^2+1\right)^{3/2}} - \frac{1}{2}\,\frac{2\,y+2}{\left((x-1)^2+(y+1)^2+1\right)^{3/2}} - \frac{1}{2}\,\frac{2\,y+2}{\left((x+1)^2+(y+1)^2+1\right)^{3/2}}\right]\right] \quad \textbf{(2.5)}$$

```
> fieldplot(gr, x = -2 .. 2, y = -2.3 .. 2.3, axes = frame,
  thickness = 2);
```



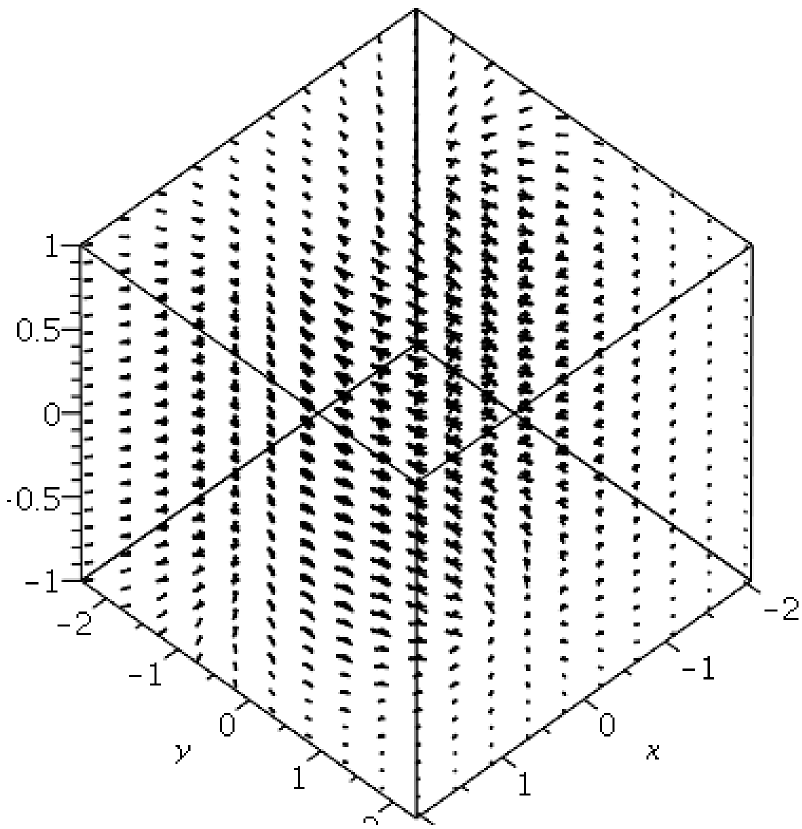

```
> divgr := Divergence(gr):
> plot3d(divgr,x=-2..2 ,y=-2..2,lightmodel=none,color=divgr);
```



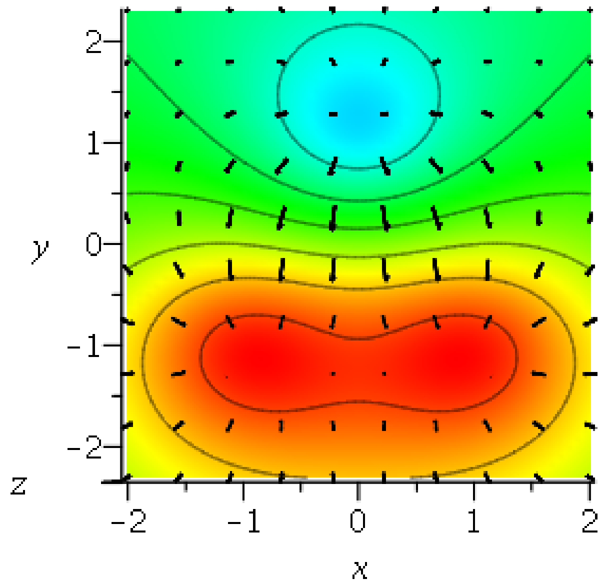

```
> gr3 := Gradient(k, [x,y,z]);
```

$$gr3 := \left[\left[\frac{x}{(x^2+(y-1)^2+1)^{3/2}} - \frac{1}{2}\frac{2x-2}{((x-1)^2+(y+1)^2+1)^{3/2}} - \frac{1}{2}\frac{2x+2}{((x+1)^2+(y+1)^2+1)^{3/2}}\right], \left[\frac{1}{2}\frac{2y-2}{(x^2+(y-1)^2+1)^{3/2}} - \frac{1}{2}\frac{2y+2}{((x-1)^2+(y+1)^2+1)^{3/2}} - \frac{1}{2}\frac{2y+2}{((x+1)^2+(y+1)^2+1)^{3/2}}\right], \left[0\right]\right] \quad (2.6)$$

```
> f3 := fieldplot3d(gr3, x = -2 .. 2, y = -2.3 .. 2.3, z = -1 ..
  1, color = black, grid = [10, 10, 20]):
> f3;
```




```
> pl3 := plot3d(k, x = -2 .. 2, y = -2.3 .. 2.3, shading = zhue,
  style = patchcontour,lightmodel=none, numpoints = 3000):
> display({f3, pl3}, axes = frame, orientation = [-90,0]);
```



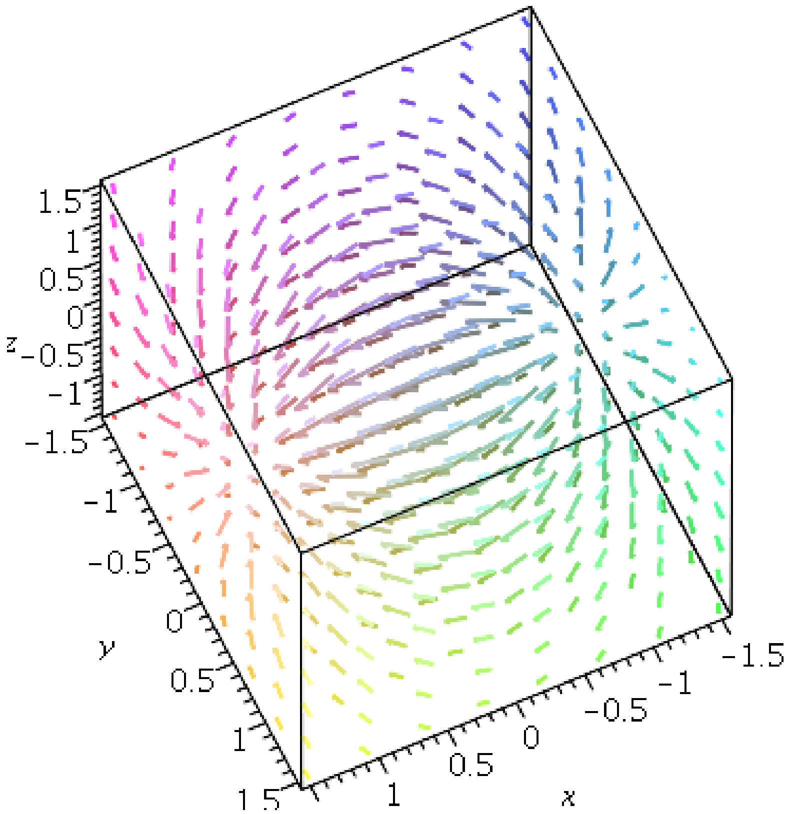

```
> k3 := 1/sqrt((x-1)^2 + y^2 + z^2 + 1) - 1/sqrt((x+1)^2 + y^2 +
  z^2 + 1);
```

$$k3 := \frac{1}{\sqrt{(x-1)^2+y^2+z^2+1}} - \frac{1}{\sqrt{(x+1)^2+y^2+z^2+1}} \quad \textbf{(2.7)}$$

```
> gr := Gradient(k3, [x,y,z]);
```

$$gr := \left[\begin{array}{c} -\frac{1}{2}\frac{2x-2}{\left((x-1)^2+y^2+z^2+1\right)^{3/2}} + \frac{1}{2}\frac{2x+2}{\left((x+1)^2+y^2+z^2+1\right)^{3/2}} \\ -\frac{y}{\left((x-1)^2+y^2+z^2+1\right)^{3/2}} + \frac{y}{\left((x+1)^2+y^2+z^2+1\right)^{3/2}} \\ -\frac{z}{\left((x-1)^2+y^2+z^2+1\right)^{3/2}} + \frac{z}{\left((x+1)^2+y^2+z^2+1\right)^{3/2}} \end{array}\right] \quad \textbf{(2.8)}$$

```
> fieldplot3d(gr, x = -1.5..1.5, y = -1.5..1.5, z = -1.5..1.5,
  orientation = [65, 30], axes = boxed, thickness = 2);
```



Jacobimatrix

```
> restart:
> with(VectorCalculus):
> BasisFormat(false):
> F := <F1(x,y,z), F2(x,y,z)>;
```

$$F := \begin{bmatrix} F1(x, y, z) \\ F2(x, y, z) \end{bmatrix} \quad (3.1)$$

```
> Jacobian(F, [x,y,z]);
```

$$\begin{bmatrix} \frac{\partial}{\partial x} F1(x, y, z) & \frac{\partial}{\partial y} F1(x, y, z) & \frac{\partial}{\partial z} F1(x, y, z) \\ \frac{\partial}{\partial x} F2(x, y, z) & \frac{\partial}{\partial y} F2(x, y, z) & \frac{\partial}{\partial z} F2(x, y, z) \end{bmatrix} \quad (3.2)$$

```
> F3 := <F[1], F[2], 0>;
```

$$F3 := \begin{bmatrix} F1(x,y,z) \\ F2(x,y,z) \\ 0 \end{bmatrix} \tag{3.3}$$

```
> Jacobian(F3, [x,y,z]);
```

$$\begin{bmatrix} \frac{\partial}{\partial x} F1(x,y,z) & \frac{\partial}{\partial y} F1(x,y,z) & \frac{\partial}{\partial z} F1(x,y,z) \\ \frac{\partial}{\partial x} F2(x,y,z) & \frac{\partial}{\partial y} F2(x,y,z) & \frac{\partial}{\partial z} F2(x,y,z) \\ 0 & 0 & 0 \end{bmatrix} \tag{3.4}$$

```
> with(LinearAlgebra):
> jac := SubMatrix(%%, 1..2, 1..3);
```

$$jac := \begin{bmatrix} \frac{\partial}{\partial x} F1(x,y,z) & \frac{\partial}{\partial y} F1(x,y,z) & \frac{\partial}{\partial z} F1(x,y,z) \\ \frac{\partial}{\partial x} F2(x,y,z) & \frac{\partial}{\partial y} F2(x,y,z) & \frac{\partial}{\partial z} F2(x,y,z) \end{bmatrix} \tag{3.5}$$

```
> F := <x^2 + 2*x + 2 + y^2 - 2*y, x^2 + 2*x - y^2 + 2*y, x*y - x
  + y -1>;
```

$$F := \begin{bmatrix} x^2 + y^2 + 2x - 2y + 2 \\ x^2 - y^2 + 2x + 2y \\ xy - x + y - 1 \end{bmatrix} \tag{3.6}$$

```
> Jacobian(F, [x,y,z]);
```

$$\begin{bmatrix} 2x+2 & 2y-2 & 0 \\ 2x+2 & -2y+2 & 0 \\ y-1 & x+1 & 0 \end{bmatrix} \tag{3.7}$$

```
> J := SubMatrix(%, 1..3, 1..2);
```

$$J := \begin{bmatrix} 2x+2 & 2y-2 \\ 2x+2 & -2y+2 \\ y-1 & x+1 \end{bmatrix} \tag{3.8}$$

```
> Rank(J); # Vorsicht falsch
```

$$2 \tag{3.9}$$

```
> ReducedRowEchelonForm(J);
```

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{3.10}$$

```
> J;
```

$$\begin{bmatrix} 2x+2 & 2y-2 \\ 2x+2 & -2y+2 \\ y-1 & x+1 \end{bmatrix} \tag{3.11}$$

```
> J1 := RowOperation(J, [2,1], -1);
```

$$J1 := \begin{bmatrix} 2x+2 & 2y-2 \\ 0 & -4y+4 \\ y-1 & x+1 \end{bmatrix} \tag{3.12}$$

```
> J2 := RowOperation(J1, [3,2]);
```

$$J2 := \begin{bmatrix} 2x+2 & 2y-2 \\ y-1 & x+1 \\ 0 & -4y+4 \end{bmatrix} \tag{3.13}$$

```
> J3 := RowOperation(J2, 1, y-1);    # ausser fuer y = 1
```

$$J3 := \begin{bmatrix} (y-1)(2x+2) & (y-1)(2y-2) \\ y-1 & x+1 \\ 0 & -4y+4 \end{bmatrix} \tag{3.14}$$

```
> J4 := RowOperation(J3, 2, 2*x+2);    # ausser fuer x = -1;
```

$$J4 := \begin{bmatrix} (y-1)(2x+2) & (y-1)(2y-2) \\ (y-1)(2x+2) & (x+1)(2x+2) \\ 0 & -4y+4 \end{bmatrix} \tag{3.15}$$

```
> RowOperation(J4, [2,1], -1);
```

$$\begin{bmatrix} (y-1)(2x+2) & (y-1)(2y-2) \\ 0 & (x+1)(2x+2)-(y-1)(2y-2) \\ 0 & -4y+4 \end{bmatrix} \tag{3.16}$$

```
> J5 := map(expand, %);
```

$$J5 := \begin{bmatrix} 2xy-2x+2y-2 & 2y^2-4y+2 \\ 0 & 2x^2-2y^2+4x+4y \\ 0 & -4y+4 \end{bmatrix} \tag{3.17}$$

```
> map(factor, J5);
```

$$\begin{bmatrix} 2(x+1)(y-1) & 2(y-1)^2 \\ 0 & 2(y+x)(x+2-y) \\ 0 & -4y+4 \end{bmatrix} \tag{3.18}$$

Also ist fuer x <>-1 und y<>1 der Rang tatsaechlich 2. Wir testen den Extremfall

```
> subs(x = -1, y = 1, J);
```

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (3.19)$$

Hessematrix

f : R^n --> R

```
> with(VectorCalculus):
> f := (x, y, z) -> exp(x^2+y^2+z);
```

$$f := (x, y, z) \rightarrow e^{x^2+y^2+z} \quad (4.1)$$

```
> Hessian(f(x, y, z), [x, y, z]);
```

$$\begin{bmatrix} 2\,e^{x^2+y^2+z} + 4\,x^2\,e^{x^2+y^2+z} & 4\,x\,y\,e^{x^2+y^2+z} & 2\,x\,e^{x^2+y^2+z} \\ 4\,x\,y\,e^{x^2+y^2+z} & 2\,e^{x^2+y^2+z} + 4\,y^2\,e^{x^2+y^2+z} & 2\,y\,e^{x^2+y^2+z} \\ 2\,x\,e^{x^2+y^2+z} & 2\,y\,e^{x^2+y^2+z} & e^{x^2+y^2+z} \end{bmatrix} \quad (4.2)$$

```
> g := exp(x^2+y^2+z);
```

$$g := e^{x^2+y^2+z} \quad (4.3)$$

```
> Hessian(g, [x, y, z]);
```

$$\begin{bmatrix} 2\,e^{x^2+y^2+z} + 4\,x^2\,e^{x^2+y^2+z} & 4\,x\,y\,e^{x^2+y^2+z} & 2\,x\,e^{x^2+y^2+z} \\ 4\,x\,y\,e^{x^2+y^2+z} & 2\,e^{x^2+y^2+z} + 4\,y^2\,e^{x^2+y^2+z} & 2\,y\,e^{x^2+y^2+z} \\ 2\,x\,e^{x^2+y^2+z} & 2\,y\,e^{x^2+y^2+z} & e^{x^2+y^2+z} \end{bmatrix} \quad (4.4)$$

```
> with(LinearAlgebra):
> IsDefinite(subs([x = 1, y = 2, z = 1], (4.4)));
```

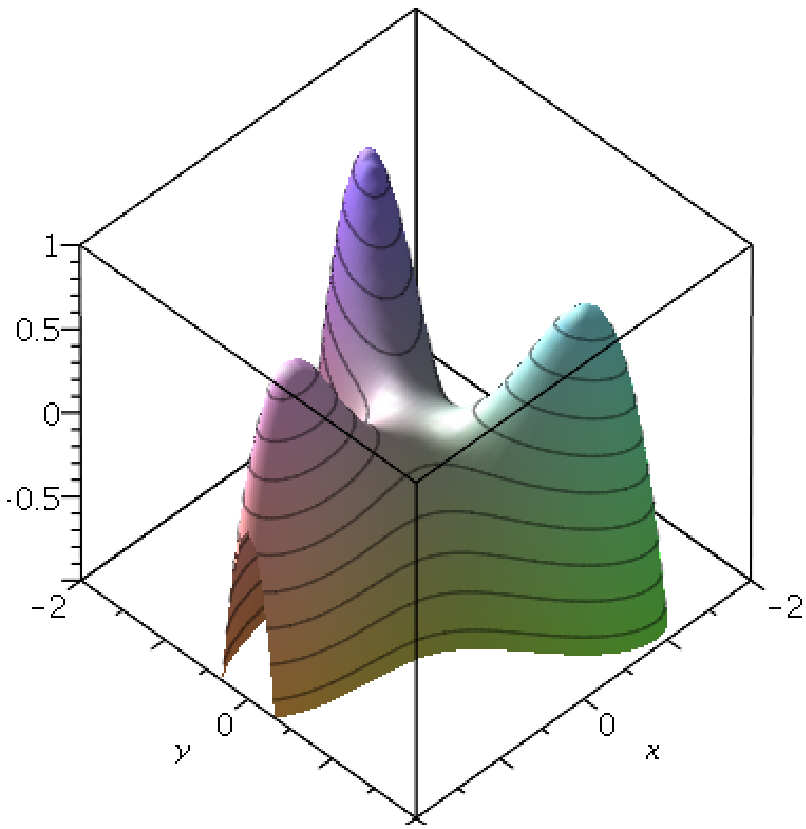
true (4.5)

Lokale Extrema

```
> restart;
> with(VectorCalculus): with(LinearAlgebra):
> f:= -1/2*x^4 - x^2*y^2-1/2*y^4+x^3-3*x*y^2;
```

$$f := -\frac{1}{2}\,x^4 - x^2\,y^2 - \frac{1}{2}\,y^4 + x^3 - 3\,x\,y^2 \quad (5.1)$$

```
> plot3d(f,x=-2..2,y=-2..2,view=-1..1,style=patchcontour);
```




```
> g:=Gradient(f,[x,y]);
```

$$g := (-2x^3 - 2xy^2 + 3x^2 - 3y^2)\bar{e}_x + (-2x^2y - 2y^3 - 6xy)\bar{e}_y \qquad (5.2)$$

```
> H:=Hessian(f,[x,y]);
```

$$H := \begin{bmatrix} -6x^2 - 2y^2 + 6x & -4xy - 6y \\ -4xy - 6y & -2x^2 - 6y^2 - 6x \end{bmatrix} \qquad (5.3)$$

```
> solve(convert(g,set),{x,y});
```

$$\{x = 0, y = 0\}, \left\{x = \frac{3}{2}, y = 0\right\}, \left\{x = -\frac{3}{4}, y = \frac{3}{4}\, RootOf(\_Z^2 - 3)\right\} \qquad (5.4)$$

```
> L:=solve(convert(g,set),{x,y},Explicit,DropMultiplicity);
```

$$L := \{x = 0, y = 0\}, \left\{x = \frac{3}{2}, y = 0\right\}, \left\{x = -\frac{3}{4}, y = \frac{3}{4}\sqrt{3}\right\}, \left\{x = -\frac{3}{4}, y = -\frac{3}{4}\sqrt{3}\right\} \qquad (5.5)$$

```
> HH := seq(subs(L[k],H),k=1..4);
```

```
> IsDefinite(HH[2]);
```

$$HH := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -\frac{9}{2} & 0 \\ 0 & -\frac{27}{2} \end{bmatrix}, \begin{bmatrix} -\frac{45}{4} & -\frac{9}{4}\sqrt{3} \\ -\frac{9}{4}\sqrt{3} & -\frac{27}{4} \end{bmatrix}, \begin{bmatrix} -\frac{45}{4} & \frac{9}{4}\sqrt{3} \\ \frac{9}{4}\sqrt{3} & -\frac{27}{4} \end{bmatrix}$$

false **(5.6)**

```
> IsDefinite(HH[2],query=negative_definite);
```

true **(5.7)**

```
> IsDefinite(HH[3],query=negative_definite);
```

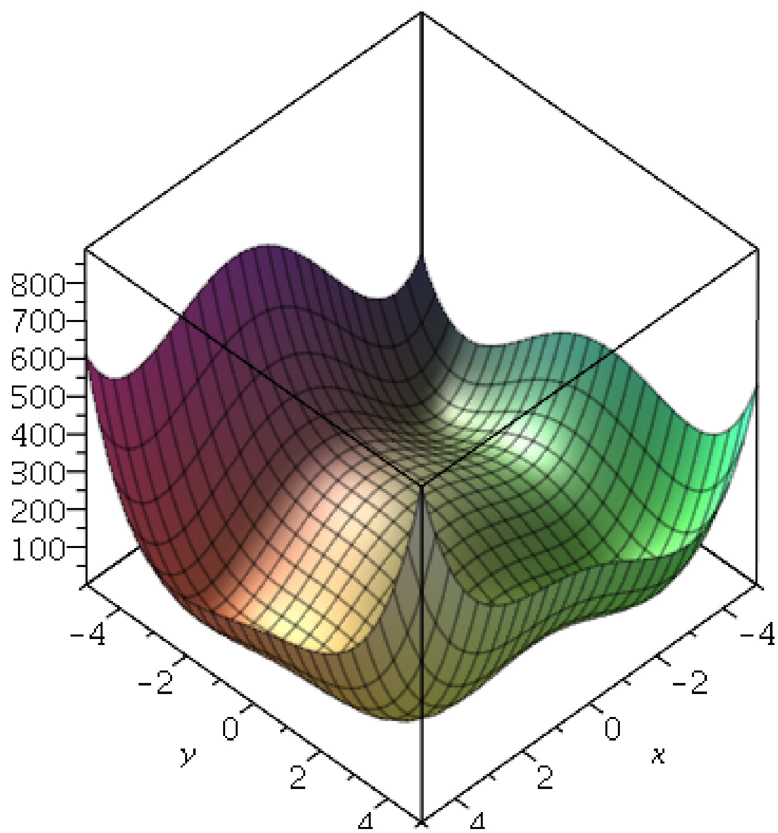
true **(5.8)**

Noch ein Beispiel

```
> f := (x^2+y-11)^2 + (x+y^2-7)^2;
```

$$f := (x^2 + y - 11)^2 + (y^2 + x - 7)^2$$ **(5.9)**

```
> plot3d(f,x=-5..5,y=-5..5); # Himmelblaufunktion
```



```
> BasisFormat(false):
> g:=Gradient(f,[x,y]);
```

$$g := \begin{bmatrix} 4\,(x^2+y-11)\,x+2\,y^2+2\,x-14 \\ 2\,x^2+2\,y-22+4\,(y^2+x-7)\,y \end{bmatrix} \tag{5.10}$$

```
> H:=map(factor,Hessian(f,[x,y]));
```

$$H := \begin{bmatrix} 12\,x^2+4\,y-42 & 4\,x+4\,y \\ 4\,x+4\,y & 12\,y^2+4\,x-26 \end{bmatrix} \tag{5.11}$$

```
> _EnvAllSolutions := true;
```

$$\_EnvAllSolutions := true \tag{5.12}$$

```
> L:=solve([g[1]=0,g[2]=0],{x,y});
```

$$L := \{x=3,\,y=2\},\ \left\{x=-RootOf(\_Z^3+2\,\_Z^2-10\,\_Z-19)^2+7,\ y=RootOf(\_Z^3+2\,\_Z^2-10\,\_Z-19)\right\},\ \left\{x=-\frac{1}{2}\,RootOf(\_Z^5-26\,\_Z^3-42\,\_Z^2+1)^4+13\,RootOf(\_Z^5-26\,\_Z^3-42\,\_Z^2+1)^2+21\,RootOf(\_Z^5-26\,\_Z^3-42\,\_Z^2+1),\ y=\frac{1}{2}\,RootOf(\_Z^5-26\,\_Z^3-42\,\_Z^2+1)\right\} \tag{5.13}$$

```
> AVL := seq(allvalues(L[k]),k=1..3);
```

$$AVL := \{x=3,\,y=2\},\ \left\{x=-\left(\frac{1}{6}\,(1268+12\,\mathrm{I}\sqrt{6303})^{1/3}+\frac{68}{3\,(1268+12\,\mathrm{I}\sqrt{6303})^{1/3}}-\frac{2}{3}\right)^2+7,\ y=\frac{1}{6}\,(1268+12\,\mathrm{I}\sqrt{6303})^{1/3}+\frac{68}{3\,(1268+12\,\mathrm{I}\sqrt{6303})^{1/3}}-\frac{2}{3}\right\},\ \left\{x=-\left(-\frac{1}{12}\,(1268+12\,\mathrm{I}\sqrt{6303})^{1/3}-\frac{34}{3\,(1268+12\,\mathrm{I}\sqrt{6303})^{1/3}}-\frac{2}{3}-\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{6}\,(1268+12\,\mathrm{I}\sqrt{6303})^{1/3}-\frac{68}{3\,(1268+12\,\mathrm{I}\sqrt{6303})^{1/3}}\right)\right)^2+7,\ y=-\frac{1}{12}\,(1268+12\,\mathrm{I}\sqrt{6303})^{1/3}-\frac{34}{3\,(1268+12\,\mathrm{I}\sqrt{6303})^{1/3}}-\frac{2}{3}-\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{6}\,(1268+12\,\mathrm{I}\sqrt{6303})^{1/3}-\frac{68}{3\,(1268+12\,\mathrm{I}\sqrt{6303})^{1/3}}\right)\right\}, \tag{5.14}$$

$$\left\{x = -\left(-\frac{1}{12}\left(1268+12\,\mathrm{I}\sqrt{6303}\right)^{1/3} - \frac{34}{3\left(1268+12\,\mathrm{I}\sqrt{6303}\right)^{1/3}} - \frac{2}{3} + \frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{6}\left(1268+12\,\mathrm{I}\sqrt{6303}\right)^{1/3} - \frac{68}{3\left(1268+12\,\mathrm{I}\sqrt{6303}\right)^{1/3}}\right)\right)^2 + 7, y = -\frac{1}{12}\left(1268+12\,\mathrm{I}\sqrt{6303}\right)^{1/3} - \frac{34}{3\left(1268+12\,\mathrm{I}\sqrt{6303}\right)^{1/3}} - \frac{2}{3} + \frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{6}\left(1268+12\,\mathrm{I}\sqrt{6303}\right)^{1/3} - \frac{68}{3\left(1268+12\,\mathrm{I}\sqrt{6303}\right)^{1/3}}\right)\right\}, \left\{x = -\frac{1}{2}\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 1)^4 + 13\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 1)^2 + 21\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 1), y = \frac{1}{2}\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 1)\right\}, \left\{x = -\frac{1}{2}\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 2)^4 + 13\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 2)^2 + 21\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 2), y = \frac{1}{2}\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 2)\right\}, \left\{x = -\frac{1}{2}\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 3)^4 + 13\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 3)^2 + 21\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 3), y = \frac{1}{2}\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 3)\right\}, \left\{x = -\frac{1}{2}\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 4)^4 + 13\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 4)^2 + 21\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 4), y = \frac{1}{2}\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 4)\right\}, \left\{x = -\frac{1}{2}\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 5)^4 + 13\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, \mathit{index} = 5)^2 + 21\,\mathit{RootOf}(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1,$$

$index = 5), y = \frac{1}{2}\, RootOf(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, index = 5)\}$

```
> seq(simplify(evalc(AVL[k])) , k=1..7);
```

$$\{x = 3, y = 2\}, \left\{x = \frac{59}{9} + \frac{8}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right) - \frac{136}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)^2, y = -\frac{2}{3} + \frac{2}{3}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)\right\}, \left\{x = -\frac{43}{9} + \frac{68}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\sqrt{3}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right) + \frac{4}{9}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right) + \frac{68}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)^2 - \frac{4}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right), y = -\frac{2}{3} + \frac{1}{3}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right) - \frac{1}{3}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\right\}, \left\{x = -\frac{43}{9} - \frac{68}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\sqrt{3}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right) - \frac{4}{9}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right) + \frac{68}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)^2 - \frac{4}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right), y = -\frac{2}{3} - \frac{1}{3}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right) - \frac{1}{3}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\right\}, \left\{x = -\frac{1}{2}\, RootOf(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, index = 1)\,(RootOf(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, index = 1)^3 - 26\, RootOf(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, index = 1) - 42), y = \frac{1}{2}\, RootOf(\_Z^5 - 26\,\_Z^3 - 42\,\_Z^2 + 1, index = 1)\right\}, \left\{x = -\frac{1}{2}\, RootOf(\_Z^5\right. \tag{5.15}$$

$$- 26\_Z^3 - 42\_Z^2 + 1, index = 2) \left(RootOf(\_Z^5 - 26\_Z^3 - 42\_Z^2 + 1, index = 2)^3 - 26\, RootOf(\_Z^5 - 26\_Z^3 - 42\_Z^2 + 1, index = 2) - 42\right), y = \frac{1}{2} RootOf(\_Z^5 - 26\_Z^3 - 42\_Z^2 + 1, index = 2)\Big\}, \Big\{x = -\frac{1}{2} RootOf(\_Z^5 - 26\_Z^3 - 42\_Z^2 + 1, index = 3) \left(RootOf(\_Z^5 - 26\_Z^3 - 42\_Z^2 + 1, index = 3)^3 - 26\, RootOf(\_Z^5 - 26\_Z^3 - 42\_Z^2 + 1, index = 3) - 42\right), y = \frac{1}{2} RootOf(\_Z^5 - 26\_Z^3 - 42\_Z^2 + 1, index = 3)\Big\}$$

```
> seq(simplify(evalc(subs(AVL[k],g))) , k=1..7);
```

$$\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix} \qquad (5.16)$$

```
> L1:=L[1];
```

$$L1 := \{x = 3, y = 2\} \qquad (5.17)$$

```
> L2_ := allvalues(L[2]);
```

$$L2\_ := \Big\{x = -\left(\frac{1}{6}\left(1268 + 12\,I\sqrt{6303}\right)^{1/3} + \frac{68}{3\left(1268 + 12\,I\sqrt{6303}\right)^{1/3}} - \frac{2}{3}\right)^2 + 7, y = \frac{1}{6}\left(1268 + 12\,I\sqrt{6303}\right)^{1/3} + \frac{68}{3\left(1268 + 12\,I\sqrt{6303}\right)^{1/3}} - \frac{2}{3}\Big\}, \Big\{x = -\Big(-\frac{1}{12}\left(1268 + 12\,I\sqrt{6303}\right)^{1/3} - \frac{34}{3\left(1268 + 12\,I\sqrt{6303}\right)^{1/3}} - \frac{2}{3} - \frac{1}{2}\,I\sqrt{3}\left(\frac{1}{6}\left(1268 + 12\,I\sqrt{6303}\right)^{1/3} - \frac{68}{3\left(1268 + 12\,I\sqrt{6303}\right)^{1/3}}\right)\Big)^2 + 7, y = -\frac{1}{12}\left(1268 + 12\,I\sqrt{6303}\right)^{1/3} - \frac{34}{3\left(1268 + 12\,I\sqrt{6303}\right)^{1/3}} - \frac{2}{3} - \frac{1}{2}\,I\sqrt{3}\left(\frac{1}{6}\left(1268 + 12\,I\sqrt{6303}\right)^{1/3} - \frac{68}{3\left(1268 + 12\,I\sqrt{6303}\right)^{1/3}}\right)\Big\}, \Big\{x = -\Big(-\frac{1}{12}\left(1268 + 12\,I\sqrt{6303}\right)^{1/3} - \frac{34}{3\left(1268 + 12\,I\sqrt{6303}\right)^{1/3}} - \frac{2}{3} + \frac{1}{2}\,I\sqrt{3}\left(\frac{1}{6}\left(1268 + 12\,I\sqrt{6303}\right)^{1/3} \right. \qquad (5.18)$$

$$- \frac{68}{3\left(1268+12\,\mathrm{I}\sqrt{6303}\right)^{1/3}}\Bigg)\Bigg)^2 + 7,\ y = -\frac{1}{12}\left(1268+12\,\mathrm{I}\sqrt{6303}\right)^{1/3}$$
$$- \frac{34}{3\left(1268+12\,\mathrm{I}\sqrt{6303}\right)^{1/3}} - \frac{2}{3} + \frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{6}\left(1268+12\,\mathrm{I}\sqrt{6303}\right)^{1/3}\right.$$
$$\left.- \frac{68}{3\left(1268+12\,\mathrm{I}\sqrt{6303}\right)^{1/3}}\right)\Bigg\}$$

```
> L2[1] := {simplify(evalc(L2_[1][1])), simplify(evalc(L2_[1][2])
  )};
> L2[2] := {simplify(evalc(L2_[2][1])), simplify(evalc(L2_[2][2])
  )};
```

$$L2_1 := \left\{x = \frac{59}{9} + \frac{8}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)\right.$$
$$- \frac{136}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)^2,\ y = -\frac{2}{3}$$
$$\left.+ \frac{2}{3}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)\right\}$$

$$L2_2 := \left\{x = -\frac{43}{9}\right. \tag{5.19}$$
$$+ \frac{68}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\sqrt{3}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\right.\right.$$
$$\left.\left.\sqrt{2101}\right)\right) + \frac{4}{9}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)$$
$$+ \frac{68}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)^2$$
$$- \frac{4}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right),\ y = -\frac{2}{3}$$
$$+ \frac{1}{3}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)$$
$$\left.- \frac{1}{3}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\right\}$$

```
> L2[3] := {simplify(evalc(L2_[2][1])), simplify(evalc(L2_[2][2])
  )};
```

$$L2_3 := \left\{x = -\frac{43}{9}\right. \tag{5.20}$$
$$+ \frac{68}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\sqrt{3}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\right.\right.$$
$$\left.\left.\sqrt{2101}\right)\right) + \frac{4}{9}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)$$

$$+\frac{68}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)^2$$

$$-\frac{4}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right), y=-\frac{2}{3}$$

$$+\frac{1}{3}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)$$

$$-\frac{1}{3}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\}$$

```
> L3 := evalf(allvalues(L[3]));
```

$$L3 := \{x=3.385154184, y=0.07385187985\}, \{x=0.0866778, y=2.884254701\}, \{x=-3.073025752, y=-0.08135304430\}, \{x=-0.27084459, y=-0.9230385565\}, \{x=-0.12796136, y=-1.953714980\} \quad \textbf{(5.21)}$$

```
> subs(L[1],H);
```

$$\begin{bmatrix} 74 & 20 \\ 20 & 34 \end{bmatrix} \quad \textbf{(5.22)}$$

```
> IsDefinite((5.22));
```

true **(5.23)**

```
> subs(L2[1],H);
```

$$\left[\left[12\left(\frac{59}{9}+\frac{8}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)\right.\right.\right. \quad \textbf{(5.24)}$$

$$\left.-\frac{136}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)^2\right)^2-\frac{134}{3}$$

$$+\frac{8}{3}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right), \frac{212}{9}$$

$$+\frac{56}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)$$

$$\left.-\frac{544}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)^2\right],$$

$$\left[\frac{212}{9}+\frac{56}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)\right.$$

$$-\frac{544}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)^2, 12\left(-\frac{2}{3}\right.$$

$$\left.+\frac{2}{3}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)\right)^2+\frac{2}{9}$$

$$+\frac{32}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)$$

$$-\frac{544}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{6303}\right)\right)^2\Bigg]\Bigg]$$

```
> IsDefinite((5.24));
```

$$true \tag{5.25}$$

```
> subs(L2[2],H);
```

$$\Bigg[\Bigg[12\left(-\frac{43}{9}\right. \tag{5.26}$$

$$+\frac{68}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\sqrt{3}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\right.\right.$$

$$\left.\left.\sqrt{2101}\right)\right)+\frac{4}{9}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)$$

$$+\frac{68}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)^2$$

$$\left.-\frac{4}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\right)^2-\frac{134}{3}$$

$$+\frac{4}{3}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)$$

$$-\frac{4}{3}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right),-\frac{196}{9}$$

$$+\frac{272}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\sqrt{3}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\right.\right.$$

$$\left.\left.\sqrt{2101}\right)\right)+\frac{28}{9}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)$$

$$+\frac{272}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)^2$$

$$-\frac{28}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\Bigg],$$

$$\Bigg[-\frac{196}{9}$$

$$+\frac{272}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\sqrt{3}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\right.\right.$$

$$\left.\left.\sqrt{2101}\right)\right)+\frac{28}{9}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)$$

$$+\frac{272}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)^2$$

$$-\frac{28}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right),12\left(-\frac{2}{3}\right.$$

$$+\frac{1}{3}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)$$
$$-\frac{1}{3}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\Big)^2-\frac{406}{9}$$
$$+\frac{272}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\sqrt{3}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)+\frac{16}{9}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)$$
$$+\frac{272}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)^2$$
$$-\frac{16}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\Big]\Big]$$

```
> IsDefinite((5.26));
```

true **(5.27)**

```
> subs(L2[3],H);
```

$$\Big[\Big[12\Big(-\frac{43}{9}$$ **(5.28)**
$$+\frac{68}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\sqrt{3}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)+\frac{4}{9}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)$$
$$+\frac{68}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)^2$$
$$-\frac{4}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\Big)^2-\frac{134}{3}$$
$$+\frac{4}{3}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)$$
$$-\frac{4}{3}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right),-\frac{196}{9}$$
$$+\frac{272}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\sqrt{3}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)+\frac{28}{9}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)$$
$$+\frac{272}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)^2$$
$$-\frac{28}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\Big],$$

$$\left[-\frac{196}{9} + \frac{272}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\sqrt{3}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right) + \frac{28}{9}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right) + \frac{272}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)^2 - \frac{28}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right), 12\left(-\frac{2}{3} + \frac{1}{3}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right) - \frac{1}{3}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\right)^2 - \frac{406}{9} + \frac{272}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\sqrt{3}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right) + \frac{16}{9}\sqrt{3}\sqrt{34}\sin\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right) + \frac{272}{9}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)^2 - \frac{16}{9}\sqrt{34}\cos\left(\frac{1}{3}\arctan\left(\frac{3}{317}\sqrt{3}\sqrt{2101}\right)\right)\right]\right]$$

```
> IsDefinite((5.28));
```

true **(5.29)**

```
> seq(IsDefinite(subs(L3[k],H),query=negative_definite),k=1..5);
```

false, false, false, true, false **(5.30)**

```
> seq(IsDefinite(subs(L3[k],H),query=positive_definite),k=1..5);
```

false, false, false, false, false **(5.31)**

```
> seq(IsDefinite(subs(L3[k],H),query=positive_semidefinite),k=1..5);
```

false, false, false, false, false **(5.32)**

```
> seq(IsDefinite(subs(L3[k],H),query=negative_semidefinite),k=1..5);
```

false, false, false, true, false **(5.33)**

```
> seq(IsDefinite(subs(L3[k],H),query=indefinite),k=1..5);
```

true, true, true, false, true **(5.34)**