Computergestuetzte Mathematik zur Analysis

Lektion 14 (30. Januar)

```
> restart:
```

Gewöhnliche Differentialgleichungen II

```
> os := diff(y(x),x$2) + y(x);
> dsolve(os=0,y(x));
> dsolve({os=0,y(0)=1,D(y)(0)=0},y(x));
> l1 := rhs(??);
> gos:= diff(y(x),x$2) + 1/5*diff(y(x),x) + y(x);
> dsolve({gos=0,y(0)=1,D(y)(0)=0},y(x));
> l2 := rhs(??);
> plot([l1,l2],x=0..20);
```

Inhomogene Gewöhnliche Differentialgleichungen

```
> l3:=rhs(dsolve({os=cos(1*x),y(0)=1,D(y)(0)=0},y(x)));
> plot(l3,x=0..100); # Resonanzfall
> l4:= rhs(dsolve({os=sin(3/4*x),y(0)=1,D(y)(0)=0},y(x)));
> l5:= rhs(dsolve({os=sin(7/8*x),y(0)=1,D(y)(0)=0},y(x)));
> plot([l4,l5],x=0..100,color=[red,blue]);
```

Bessel Funktionen

```
> with(VectorCalculus):
> SetCoordinates(polar);
> Laplacian(u(r,phi),[r,phi]);
> LG:=Laplacian(v(r)*w(phi),[r,phi])+v(r)*w(phi);
> isolate(expand(LG*r^2/v(r)/w(phi)),r);
> collect(lhs(??)*v(r)-n^2*v(r),v(r));
> g:= x^2 * diff(y(x),x$2)+x*diff(y(x),x) + (x^2-n^2)*y(x);
> dsolve(g=0,y(x));
> farben:=[red,blue,cyan,magenta];
> plot([seq(BesselJ(n,x),n=0..3)],x=0..15,color=farben);
> plot([seq(BesselY(n,x),n=0..3)],x=0..15,y=-1..0.6,color=farben)
  ;
> ns1:= seq(fsolve(BesselJ(1,x)=0,x,3.5+3*(k-1)..3.5+3*k),k=1..4)
```

```
;
```

```
> fnm1 := [ r*cos(s), r*sin(s), BesselJ(1,ns1[2]*r)*cos(s)];
> plotarg := style=patchcontour,orientation=[120,60];
> plot3d(fnm1,r=0..1,s=0..2*Pi,plotarg);
> ns2 := fsolve(BesselJ(2,x)=0,x,8..9);
> fnm:= [ r*cos(s), r*sin(s), BesselJ(2,ns2*r)*cos(2*s)];
> plot3d(fnm,r=0..1,s=0..2*Pi,plotarg);
> fnm:= [ r*cos(s), r*sin(s), sin(2*k*Pi/21)*BesselJ(2,ns2*r)*cos
  (2*s)];
> with(plots):
> animate(plot3d,[fnm,r=0..1,s=0..2*Pi,plotarg],k=0..20);
```

Differentialgleichungssysteme

```
> restart;
> with(LinearAlgebra):
> A:=<<0|1|0>,<-1|0|1>,<0|0|2>>;
> T:=MatrixExponential(A,t);
> #Loesung y' = A*y , y(0) = <a,b,c>
> y0 := <a,b,c>;
> y(t) := T.y0;
> with(VectorCalculus):
> BasisFormat(false):
> diff(y(t),t) - A.y(t);
> simplify(??);
> eval(y(t),t=0);
```

Das Pendel

```
> restart;
> Dgl := diff(y(t),t$2)= -sin(y(t));
> AW:= y(0)=Pi/8,D(y)(0)=0;
> dsolve({Dgl,AW},y(t));
> Lsg:=dsolve({Dgl,AW},y(t),type=numeric,output=listprocedure);
> yl := eval(y(t),Lsg);
> yl(1);
> Dgl_os := diff(y(t),t$2) = -y(t);

> dsolve({Dgl_os,AW},y(t));
> yl_os:=unapply(rhs(??),t);
> plot([yl,yl_os],0..50,color=[black,red]);
> AW2:= y(0)=Pi/4,D(y)(0)=0;
> Lsg:=dsolve({Dgl,AW2},y(t),type=numeric,output=listprocedure);
```

```
> yl := eval(y(t),Lsg);
> dsolve({Dgl_os,AW2},y(t));
> yl_os:=unapply(rhs(??),t);
> plot([yl,yl_os],0..50,color=[black,red]);
```