Computergestuetzte Mathematik (Lineare Algebra mit Maple)

Lektion 7 (28. Nov.)

Vektoren und Matrizen

```
> x := <1,2,3>;
```

$$x := \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad (1.1)$$

```
> y := <4|5|6>;
```

$$y := \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \quad (1.2)$$

```
> A := << 1 | 2 | 3 >,
        < 4 | 5 | 6 >,
        < 7 | 8 | 9 >>;
```

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad (1.3)$$

```
> about(A);
Matrix(3, 3, [[1,2,3],[4,5,6],[7,8,9]]):
  property aliased to Matrix(3, 3, [[1,2,3],[4,5,6],[7,8,9]])
```

```
> about(x);
Vector(3, [1,2,3]):
  property aliased to Vector(3, [1,2,3])
```

```
> about(y);
Vector[row](3, [4,5,6]):
  property aliased to Vector[row](3, [4,5,6])
```

```
> AA := Matrix(3, 3, [[1, 2, 3], [4, 5, 6], [7, 8, 9]]); # alternative Eingabe
```

$$AA := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad (1.4)$$

```
> xx := Vector(3,[1,2,3]);
```

$$xx := \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad (1.5)$$

```
> A*x;
Error, (in rtable/Product) invalid arguments
> A . x; # Matrix Vektor Produkt
```

$$\begin{bmatrix} 14 \\ 32 \\ 50 \end{bmatrix} \quad (1.6)$$

```
> y . A; # Vektor Matrix Produkt
```

$$\begin{bmatrix} 66 & 81 & 96 \end{bmatrix} \quad (1.7)$$

```
> B := < < 1, 4, 7 > | < 2, 5, 8 > | < 3 , 6 , 9 >>;
```

$$B := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad (1.8)$$

```
> A - B;
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.9)$$

```
> < A | B >;
```

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 2 & 3 \\ 4 & 5 & 6 & 4 & 5 & 6 \\ 7 & 8 & 9 & 7 & 8 & 9 \end{bmatrix} \quad (1.10)$$

```
> < A , B >;
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad (1.11)$$

```
> Id := A^0;
```

$$Id := 1 \quad (1.12)$$

```
> about(Id);
```

```
1:
  All numeric values are properties as well as objects.
  Their location in the property lattice is obvious,
  in this case integer.
```

```
> C := A + 2; # A + 2*IdentityMatrix Achtung Unterschied zu
  MATLAB!
```

$$C := \begin{bmatrix} 3 & 2 & 3 \\ 4 & 7 & 6 \\ 7 & 8 & 11 \end{bmatrix} \quad (1.13)$$

```
> C^(-1); # Matrixinverse;
```

$$\begin{bmatrix} \frac{29}{32} & \frac{1}{16} & -\frac{9}{32} \\ -\frac{1}{16} & \frac{3}{8} & -\frac{3}{16} \\ -\frac{17}{32} & -\frac{5}{16} & \frac{13}{32} \end{bmatrix} \quad (1.14)$$

```
> % . (A + 2);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1.15)$$

```
> (A + 2) . (1.14);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1.16)$$

```
> Matrix(3, shape = identity);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1.17)$$

```
> Matrix( <1,2,3>, shape = diagonal);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad (1.18)$$

```
> circ := Matrix(4, (i,j) -> (i-j)^3);
```

$$circ := \begin{bmatrix} 0 & -1 & -8 & -27 \\ 1 & 0 & -1 & -8 \\ 8 & 1 & 0 & -1 \\ 27 & 8 & 1 & 0 \end{bmatrix} \quad (1.19)$$

```
> circ^(-1);
```

$$\begin{bmatrix} 0 & -\frac{1}{36} & \frac{2}{9} & -\frac{1}{36} \\ \frac{1}{36} & 0 & -\frac{3}{4} & \frac{2}{9} \\ -\frac{2}{9} & \frac{3}{4} & 0 & -\frac{1}{36} \\ \frac{1}{36} & -\frac{2}{9} & \frac{1}{36} & 0 \end{bmatrix} \quad (1.20)$$

```
> hilbert := Matrix(4, (i,j) -> 1/(i+j-1));
```

$$hilbert := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{bmatrix} \quad (1.21)$$

```
> hilbert := Matrix(400, (i,j) -> 1/(i+j-1));
```

$$hilbert := \begin{bmatrix} \text{400 x 400 Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \quad (1.22)$$

```
> with(LinearAlgebra):
> Transpose(A);
```

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \quad (1.23)$$

```
> B := A;
```

(1.24)

$$B := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad (1.24)$$

```
> B[1,2] := 222;
```

$$B_{1,2} := 222 \qquad (1.25)$$

```
> B;
```

$$\begin{bmatrix} 1 & 222 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad (1.26)$$

```
> A;
```

$$\begin{bmatrix} 1 & 222 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad (1.27)$$

```
> A[1,2] := 2;
```

$$A_{1,2} := 2 \qquad (1.28)$$

```
> B := Copy(A);
```

$$B := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad (1.29)$$

```
> B[1,2] := 777;
```

$$B_{1,2} := 777 \qquad (1.30)$$

```
> B;
```

$$\begin{bmatrix} 1 & 777 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad (1.31)$$

```
> A;
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad (1.32)$$

Lineare Gleichungssysteme

```
> restart:
```

```
> g1 := x + y - z = 1;
```

$$g1 := x + y - z = 1 \tag{2.1}$$

```
> g2 := 2*x + y - 3*z = 0;
```

$$g2 := 2\,x + y - 3\,z = 0 \tag{2.2}$$

```
> g3 := x - 2*z = -1;
```

$$g3 := x - 2\,z = -1 \tag{2.3}$$

```
> solve({g1, g2, g3}, {x,y,z});
```

$$\{x = -1 + 2\,z,\ y = 2 - z,\ z = z\} \tag{2.4}$$

```
> subs(%, {g1, g2 , g3});
```

$$\{-1 = -1,\ 0 = 0,\ 1 = 1\} \tag{2.5}$$

```
> with(LinearAlgebra):
> B := GenerateMatrix( [g1, g2, g3], [x, y, z], augmented =
  true);
```

$$B := \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 1 & -3 & 0 \\ 1 & 0 & -2 & -1 \end{bmatrix} \tag{2.6}$$

```
> A := SubMatrix(B, 1..3, 1..3);
```

$$A := \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix} \tag{2.7}$$

```
> SubMatrix(B, 1..3, 4..4);
```

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \tag{2.8}$$

```
> whattype(%);
```

$$Matrix \tag{2.9}$$

```
> b := convert((2.8), Vector);
```

$$b := \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \tag{2.10}$$

```
> whattype(%);
```

$$Vector_{column} \tag{2.11}$$

```
> x := LinearSolve(A, b);
```

(2.12)

$$x := \begin{bmatrix} -1 + 2\,\_t_3 \\ 2 - \_t_3 \\ \_t_3 \end{bmatrix} \tag{2.12}$$

```
> A . x;
```

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \tag{2.13}$$

```
> ReducedRowEchelonForm(B);
```

$$\begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.14}$$

Zeilenweise Manipulation

```
> B;
```

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 1 & -3 & 0 \\ 1 & 0 & -2 & -1 \end{bmatrix} \tag{3.1}$$

```
> A1 := RowOperation(B, [2,1], -2);
```

$$A1 := \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -1 & -1 & -2 \\ 1 & 0 & -2 & -1 \end{bmatrix} \tag{3.2}$$

```
> A2 := RowOperation(A1, [3,1], -1);
```

$$A2 := \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -1 & -1 & -2 \\ 0 & -1 & -1 & -2 \end{bmatrix} \tag{3.3}$$

```
> A3 := RowOperation(A2, [3,2], -1);
```

$$A3 := \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.4}$$

```
> A4 := RowOperation(A3, [1,2], 1);
```

(3.5)

$$A4 := \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & -1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.5}$$

```
> A4 := RowOperation(A3, 2, -1);
```

$$A4 := \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.6}$$

Rang und Determinante

```
> Rank(B);
```

$$2 \tag{4.1}$$

```
> Determinant(A);
```

$$0 \tag{4.2}$$

Normalformen

```
> A; Eigenvalues(A);
```

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix} \tag{5.1}$$

```
> ew,T := Eigenvectors(A);
```

$$ew, T := \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 & 2 & 0 \\ 5 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{5.2}$$

```
> J := Matrix(ew, shape=diagonal);
```

$$J := \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix} \tag{5.3}$$

```
> T . J . T^(-1);
```

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix} \qquad (5.4)$$

```
> M := << -14 |  -18 |    3 |   11 |  -1 |   16>,
         < -28 |  -36 |   18 |   24 |  -6 |   40>,
         <-134 | -182 |   90 |  126 | -16 |  198>,
         < -12 |  -12 |    2 |   10 |  -2 |    8>,
         < 190 |  254 | -126 | -178 |  24 | -278>,
         <  46 |   62 |  -32 |  -46 |    4|  -66>>;
```

$$M := \begin{bmatrix} -14 & -18 & 3 & 11 & -1 & 16 \\ -28 & -36 & 18 & 24 & -6 & 40 \\ -134 & -182 & 90 & 126 & -16 & 198 \\ -12 & -12 & 2 & 10 & -2 & 8 \\ 190 & 254 & -126 & -178 & 24 & -278 \\ 46 & 62 & -32 & -46 & 4 & -66 \end{bmatrix} \qquad (5.5)$$

```
> J, T := JordanForm(M, output = ['J', 'Q']);
```

$$J, T := \begin{bmatrix} -4 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2\,I & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\,I & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}, \qquad (5.6)$$

$$\begin{bmatrix} 6 & -10-24\,I & -10+24\,I & -72 & 46 & 15 \\ 3 & -7+17\,I & -7-17\,I & 144 & -128 & 11 \\ 0 & \frac{41}{2}-\frac{3}{2}\,I & \frac{41}{2}+\frac{3}{2}\,I & 0 & 36 & -41 \\ 6 & -17-7\,I & -17+7\,I & 0 & -72 & 28 \\ 0 & -\frac{7}{2}+\frac{17}{2}\,I & -\frac{7}{2}-\frac{17}{2}\,I & -144 & 128 & 7 \\ 3 & -12+5\,I & -12-5\,I & 72 & -46 & 21 \end{bmatrix}$$

```
> T . J . T^(-1) - M;
```

(5.7)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5.7)$$

Andere Operationen mit Matrizen

```
> restart:
> with(LinearAlgebra):
> v := Vector(3, symbol = x, orientation = column);
```

$$v := \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (6.1)$$

```
> w := Vector(3, symbol=y, orientation = column);
```

$$w := \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad (6.2)$$

```
> w[1];
```

$$y_1 \quad (6.3)$$

```
> v . w;
```

$$\overline{x_1}\, y_1 + \overline{x_2}\, y_2 + \overline{x_3}\, y_3 \quad (6.4)$$

```
> v . w assuming real;
```

$$x_1\, y_1 + x_2\, y_2 + x_3\, y_3 \quad (6.5)$$

```
> CrossProduct(v, w);
```

$$\begin{bmatrix} x_2\, y_3 - x_3\, y_2 \\ x_3\, y_1 - x_1\, y_3 \\ x_1\, y_2 - x_2\, y_1 \end{bmatrix} \quad (6.6)$$

```
> VectorNorm(v);
```

$$\max(|x_1|, |x_2|, |x_3|) \quad (6.7)$$

```
> VectorNorm(v, 2);
```

$$\sqrt{|x_1|^2 + |x_2|^2 + |x_3|^2} \quad (6.8)$$

```
> with(plots):
> A := <<3, 1, 2>|<5, 2, 1>>;
```

$$A := \begin{bmatrix} 3 & 5 \\ 1 & 2 \\ 2 & 1 \end{bmatrix} \quad \textbf{(6.9)}$$

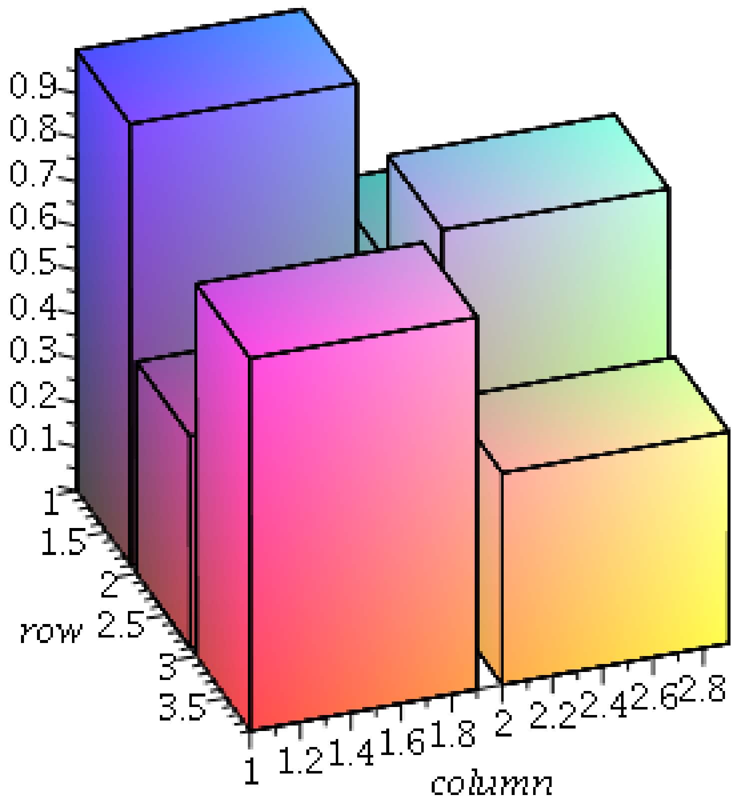
```
> map( x -> x^2, A);
```

$$\begin{bmatrix} 9 & 25 \\ 1 & 4 \\ 4 & 1 \end{bmatrix} \quad \textbf{(6.10)}$$

```
> map( x -> sin(x/2), A);
```

$$\begin{bmatrix} \sin\left(\frac{3}{2}\right) & \sin\left(\frac{5}{2}\right) \\ \sin\left(\frac{1}{2}\right) & \sin(1) \\ \sin(1) & \sin\left(\frac{1}{2}\right) \end{bmatrix} \quad \textbf{(6.11)}$$

```
> matrixplot((6.11), heights = histogram,gap=0.1,axes=frame,
  orientation=[-20,60]);
```



0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1
1.5
2
row
2.5
3
3.5
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
column