Computergestuetzte Mathematik zur Analysis

Lektion 6 (21. Nov.)

Loesen von Gleichungen (solve / fsolve)

```
> Glg := (x-1)^2 = 4-x;
```

$$Glg := (x-1)^2 = 4-x \tag{1.1}$$

```
> Lsg := solve(Glg, x);
```

$$Lsg := \frac{1}{2}+\frac{1}{2}\sqrt{13}, \frac{1}{2}-\frac{1}{2}\sqrt{13} \tag{1.2}$$

```
> subs(x = Lsg[1], Glg);
```

$$\left(-\frac{1}{2}+\frac{1}{2}\sqrt{13}\right)^2 = \frac{7}{2}-\frac{1}{2}\sqrt{13} \tag{1.3}$$

```
> subs(x = Lsg[2], Glg);
```

$$\left(-\frac{1}{2}-\frac{1}{2}\sqrt{13}\right)^2 = \frac{7}{2}+\frac{1}{2}\sqrt{13} \tag{1.4}$$

```
> simplify(op(1,(1.4))-op(2,(1.4)));
```

$$0 \tag{1.5}$$

```
> Gls := {x^2 + y^2 = 1, x = y};
```

$$Gls := \{x = y, x^2 + y^2 = 1\} \tag{1.6}$$

```
> vars := {x, y};
```

$$vars := \{x, y\} \tag{1.7}$$

```
> Lsg := solve(Gls, vars);
```

$$Lsg := \{x = RootOf(2\_Z^2 - 1, label = \_L1), y = RootOf(2\_Z^2 - 1, label = \_L1)\} \tag{1.8}$$

```
> solve(Gls,{x,y});
```

$$\{x = RootOf(2\_Z^2 - 1, label = \_L3), y = RootOf(2\_Z^2 - 1, label = \_L3)\} \tag{1.9}$$

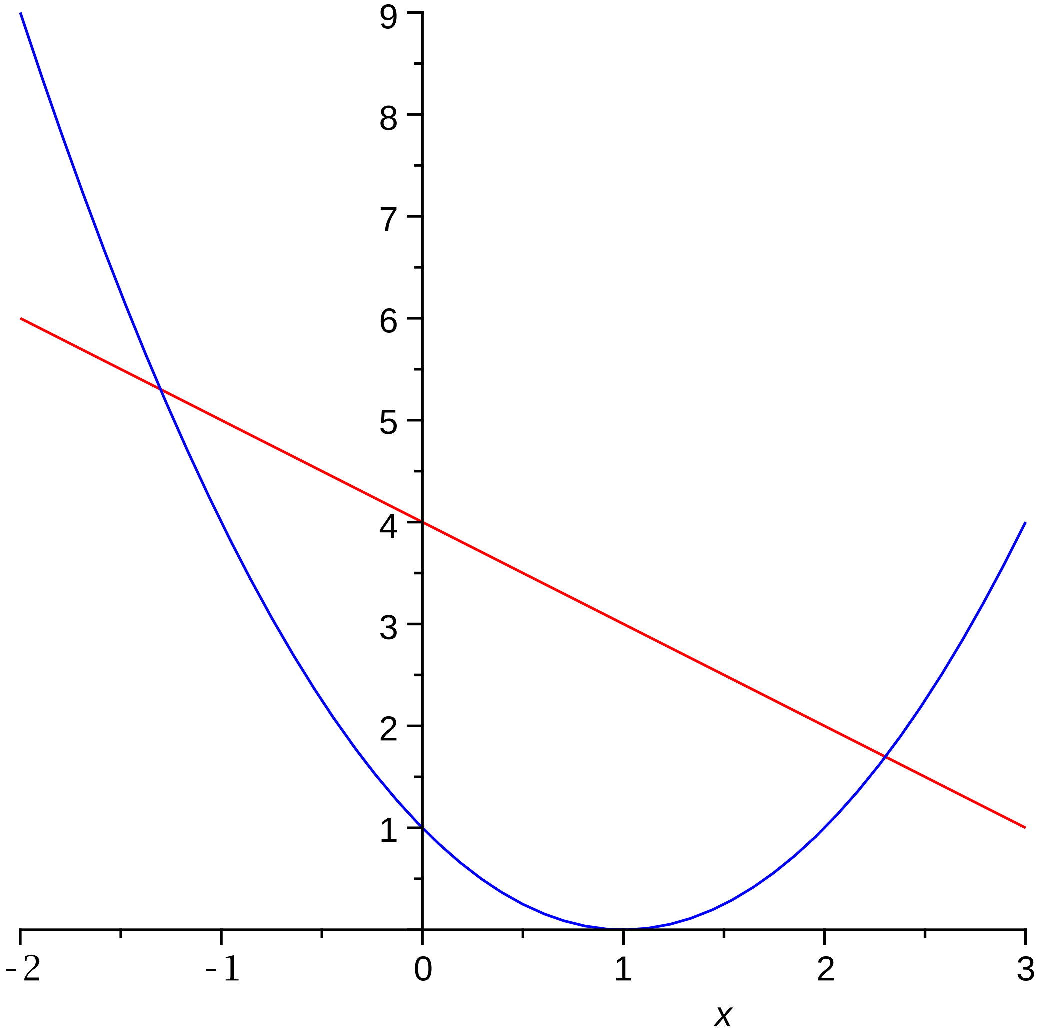
```
> allvalues(Lsg);
```

$$\left\{x = \frac{1}{2}\sqrt{2}, y = \frac{1}{2}\sqrt{2}\right\}, \left\{x = -\frac{1}{2}\sqrt{2}, y = -\frac{1}{2}\sqrt{2}\right\} \tag{1.10}$$

```
> Glg;
```

$$(x-1)^2 = 4-x \tag{1.11}$$

```
> plot([rhs(Glg),lhs(Glg)],x=-2..3,color=[red,blue]);
```




```
> solve(Glg);
```

$$\frac{1}{2}+\frac{1}{2}\sqrt{13},\ \frac{1}{2}-\frac{1}{2}\sqrt{13} \qquad (1.12)$$

```
> FLsg := fsolve(Glg,x);
```

$$FLsg := -1.302775638,\ 2.302775638 \qquad (1.13)$$

```
> subs(x = FLsg[1], Glg);
```

$$5.302775639 = 5.302775638 \qquad (1.14)$$

```
> FLsg := fsolve(Gls, vars);
```

$$FLsg := \{x = -0.7071067812,\ y = -0.7071067812\} \qquad (1.15)$$

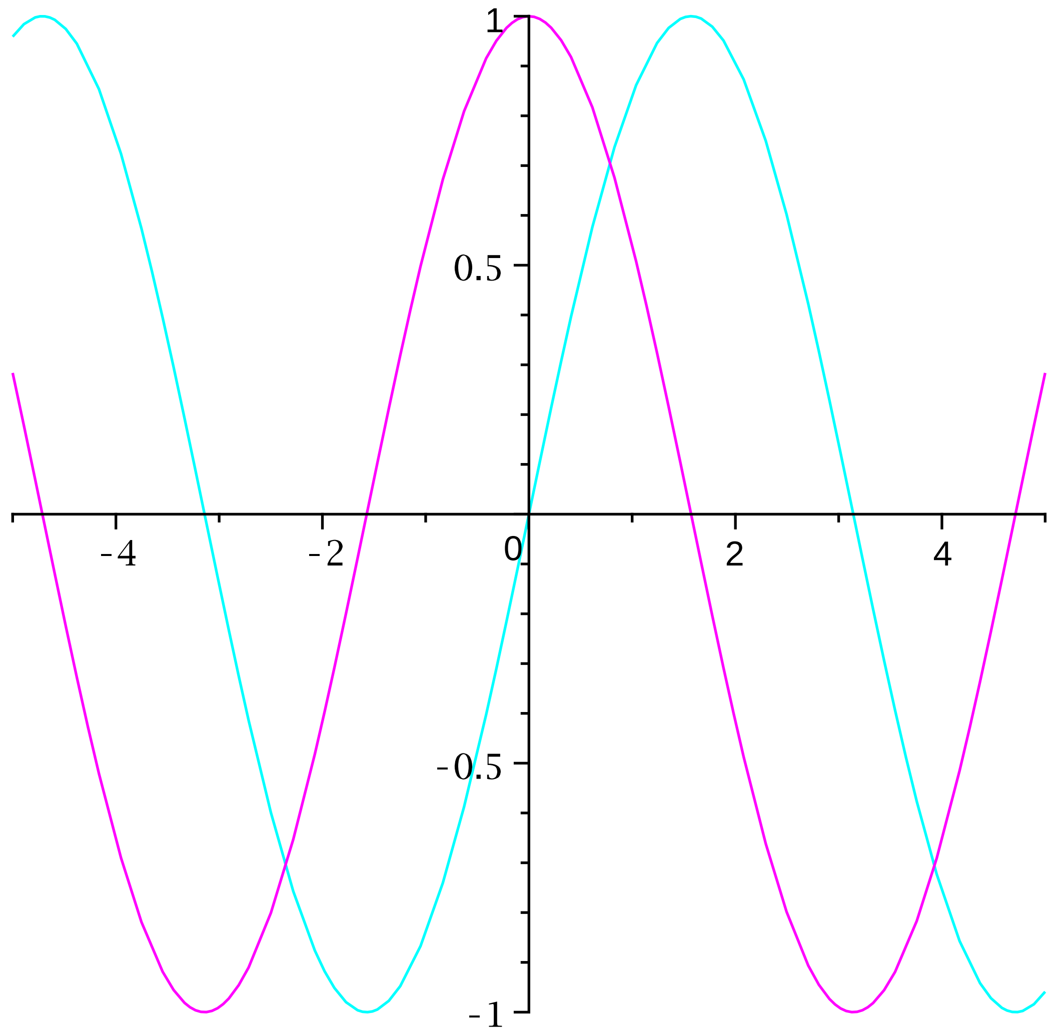
```
> FLsgA := fsolve(Gls, vars, avoid = {FLsg});
```

$$FLsgA := \{x = 0.7071067812,\ y = 0.7071067812\} \qquad (1.16)$$

```
> solve(sin(x) = cos(x), x);
```

$$\frac{1}{4}\pi \qquad (1.17)$$

```
> plot([sin,cos],-5..5,color=[cyan,magenta]);
```




```
> _EnvAllSolutions := true;  #Umgebungsvariable
```

$$\_EnvAllSolutions := true \quad (1.18)$$

```
> solve(sin(x) = cos(x), x);
```

$$\frac{1}{4}\pi + \pi\_Z1\sim \quad (1.19)$$

```
> about(_Z1);
Originally _Z1, renamed _Z1~:
  is assumed to be: integer
```

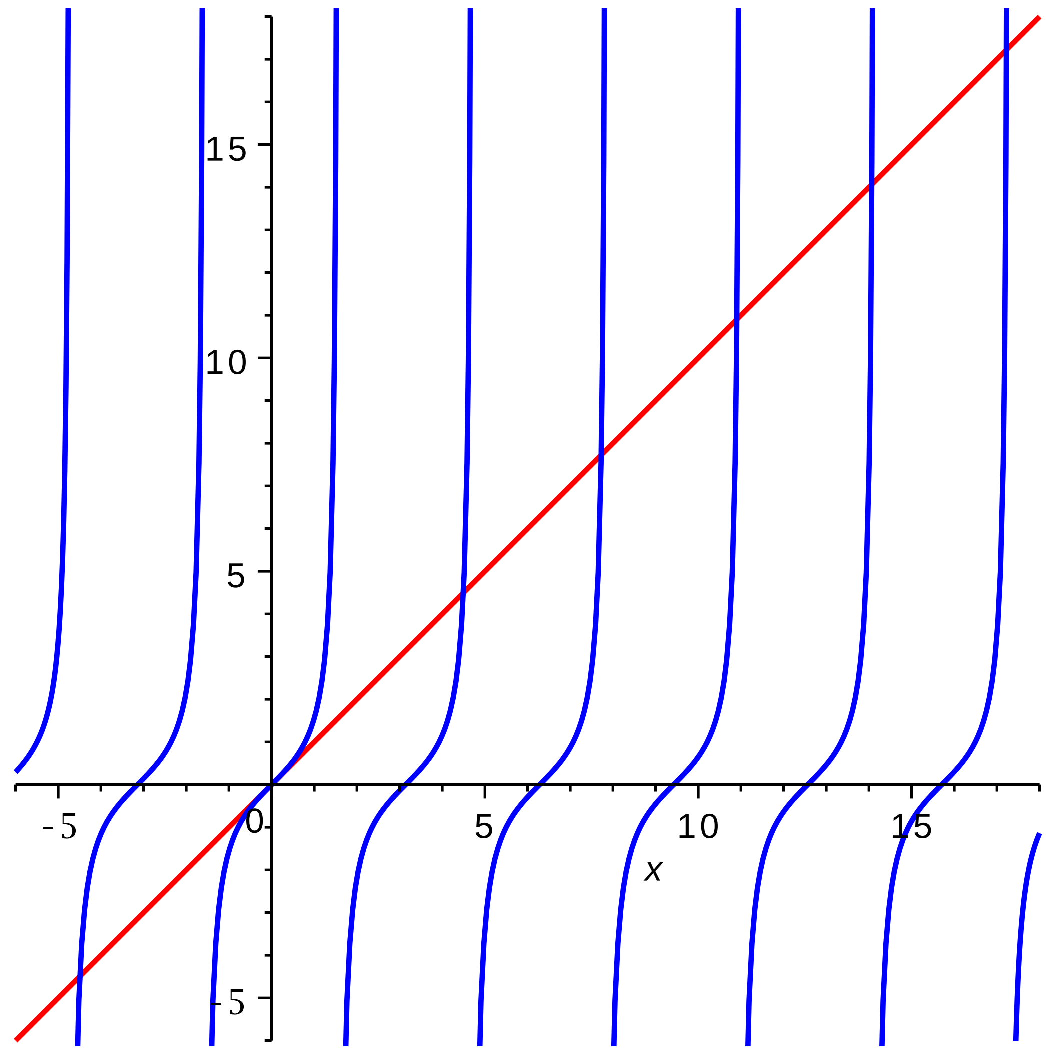
```
> _EnvAllSolutions := false;
```

$$\_EnvAllSolutions := false \quad (1.20)$$

```
> id := x -> x;
```

$$id := x \rightarrow x \quad (1.21)$$

```
> plot([id(x),tan(x)],x=-6..18, -6 .. 18, discont = true,
  thickness = 2,color=[red,blue]);
```




```
> Glg := tan(x) = x;
```

$$Glg := \tan(x) = x \quad (1.22)$$

```
> solve(Glg, x);
```

$$RootOf(-\tan(\_Z) + \_Z) \quad (1.23)$$

```
> fsolve(Glg, x);
```

$$0. \quad (1.24)$$

```
> fsolve(Glg, x, avoid = {x = 0});
```

$$-4.493409458 \quad (1.25)$$

```
> fsolve(Glg, x = 4 .. 6);
```

$$4.493409458 \quad (1.26)$$

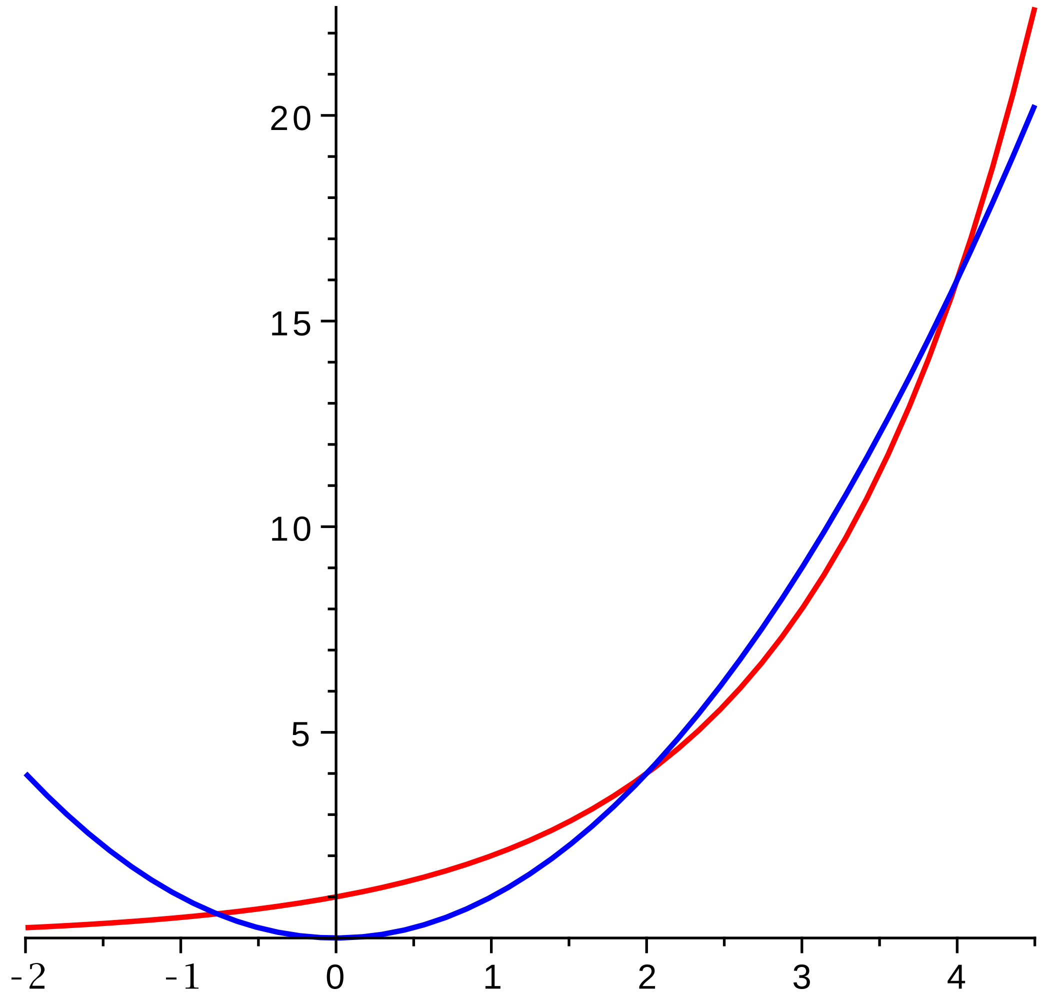
```
> f := x -> 2^x;
```

$$f := x \rightarrow 2^x \quad (1.27)$$

```
> g := x -> x^2;
```

$$g := x \rightarrow x^2 \quad (1.28)$$

```
> plot([f, g], -2 .. 4.5, thickness = 2,color=[red,blue]);
```




```
> Glg := f(x) = g(x);
```

$$Glg := 2^x = x^2 \qquad (1.29)$$

```
> solve(Glg,x);
Warning, solutions may have been lost
> evalf(%);
```

$$2.^x = x^2 \qquad (1.30)$$

Graphen von Loesungsmengen

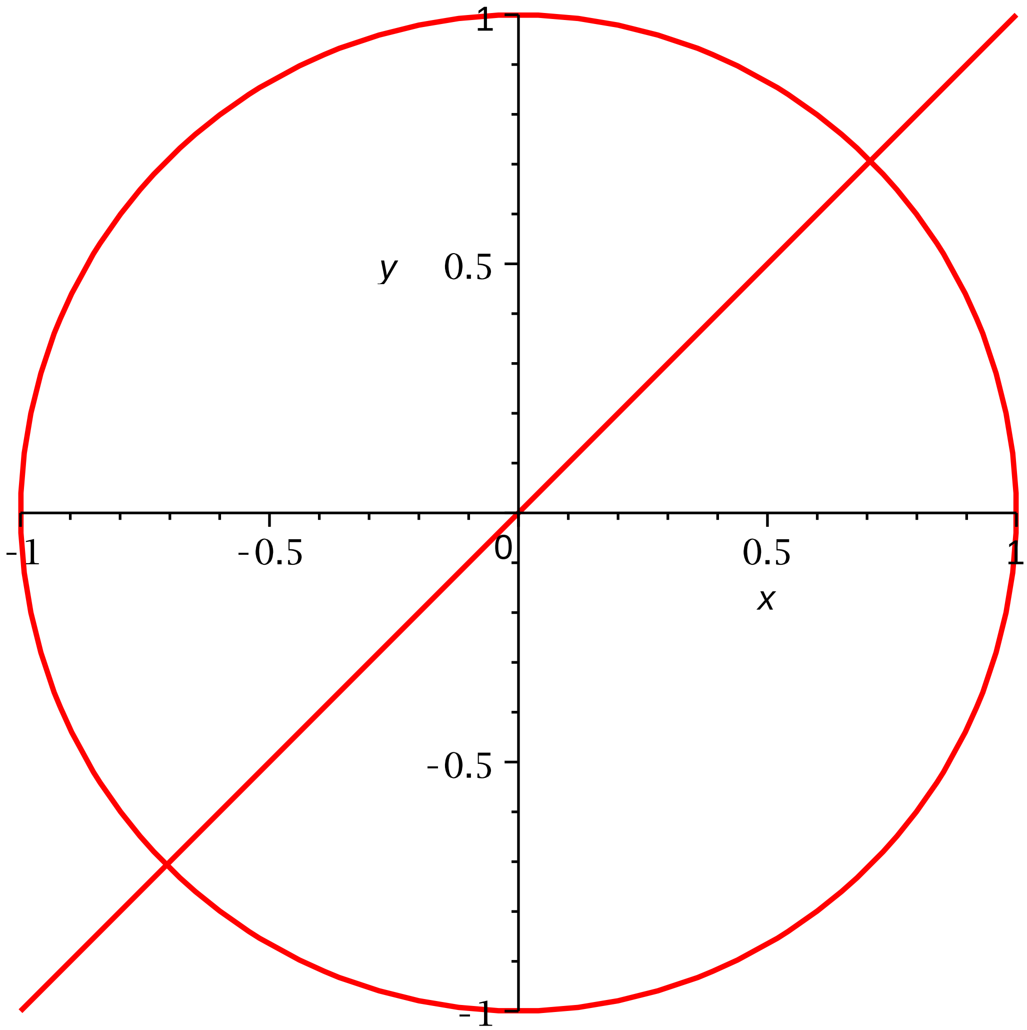
```
> with(plots);
```

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *graphplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, (2.1) *pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*]

```
> Gls := {x^2+y^2=1, x=y};
```

$$Gls := \{x = y, x^2 + y^2 = 1\} \tag{2.2}$$

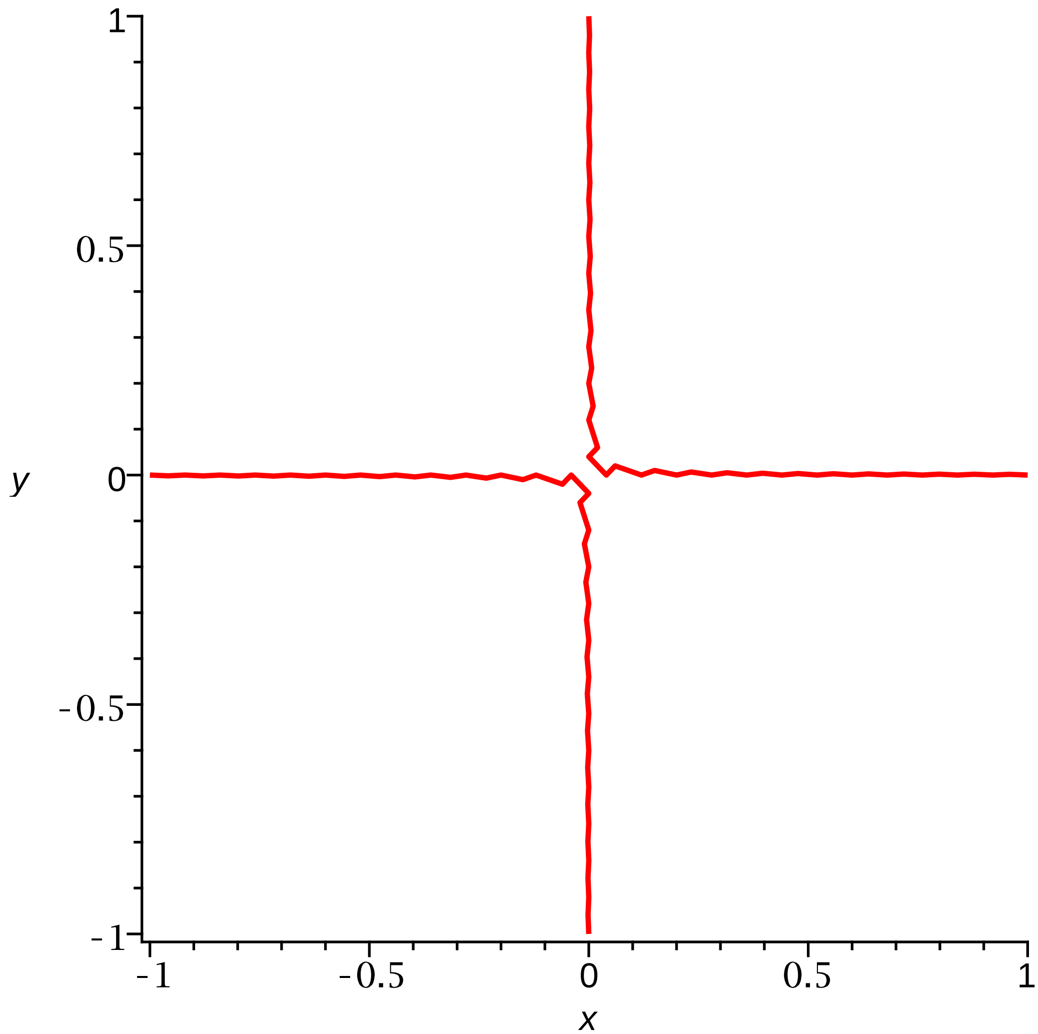
```
> implicitplot(Gls, x = -1 .. 1, y = -1 .. 1, thickness = 2,
  scaling = constrained);
```




```
> Glg := x*y = 0;
```

$$Glg := x\,y = 0 \tag{2.3}$$

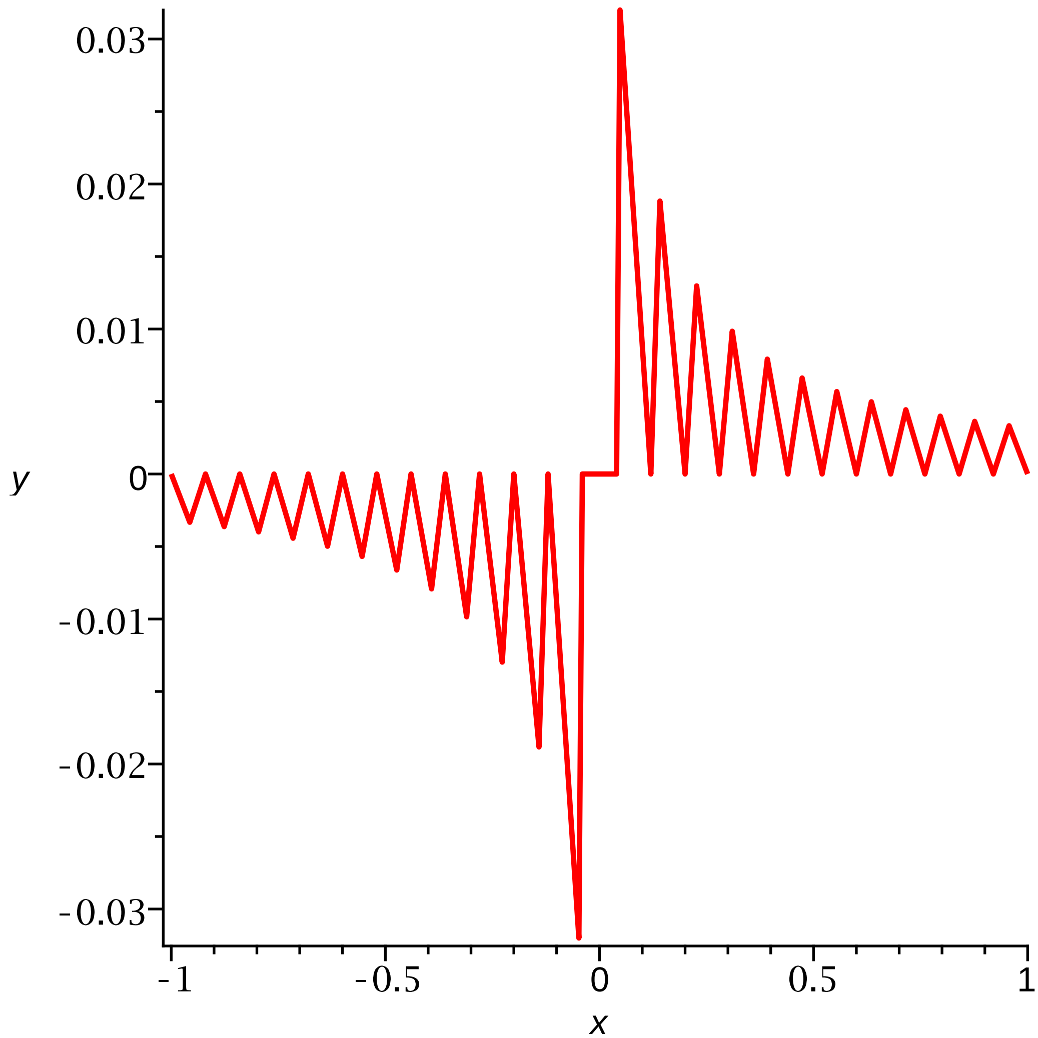
```
> implicitplot(Glg, x = -1 .. 1, y = -1 .. 1, thickness = 2,
  axes = frame);
```



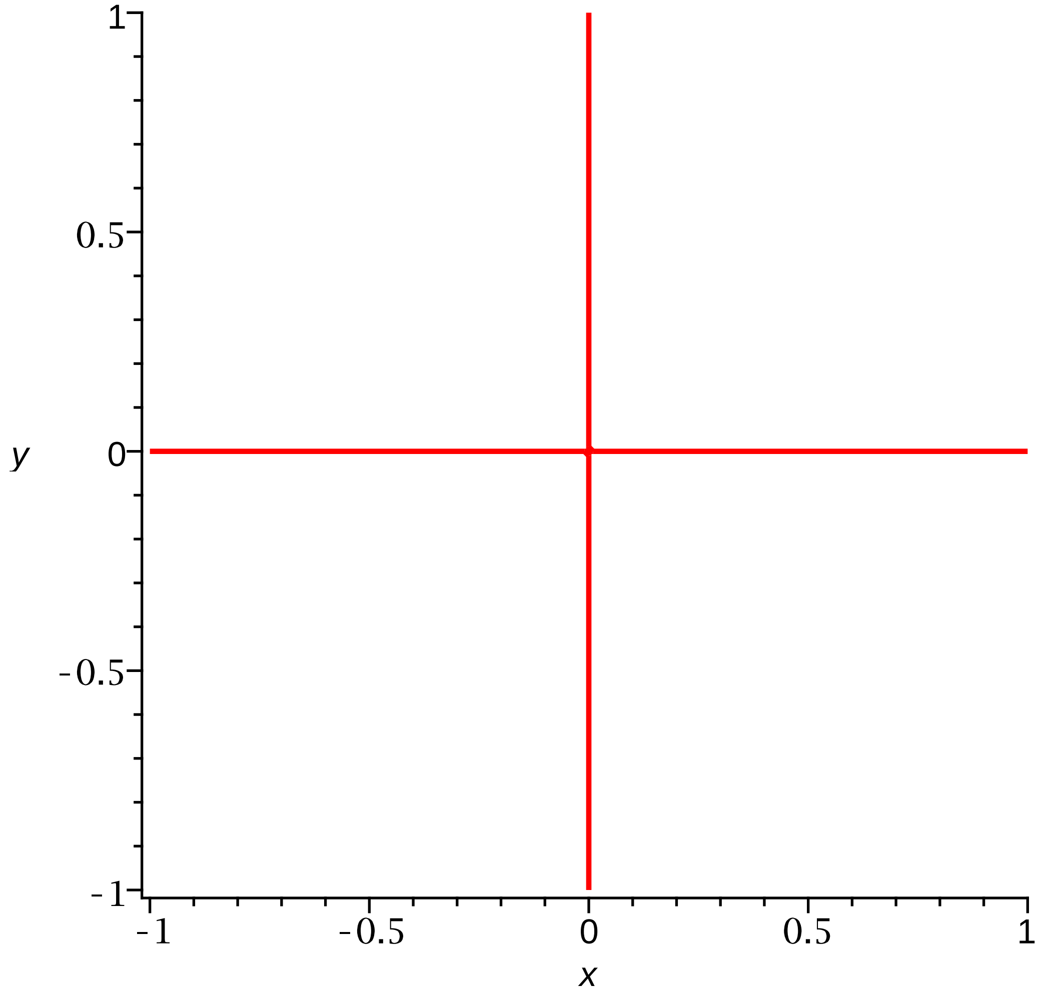

```
> Glg := x^2*y = 0;
```

$$Glg := x^2\,y = 0 \qquad (2.4)$$

```
> implicitplot(Glg, x = -1 .. 1, y = -1 .. 1, thickness = 2,
  axes = frame);
```



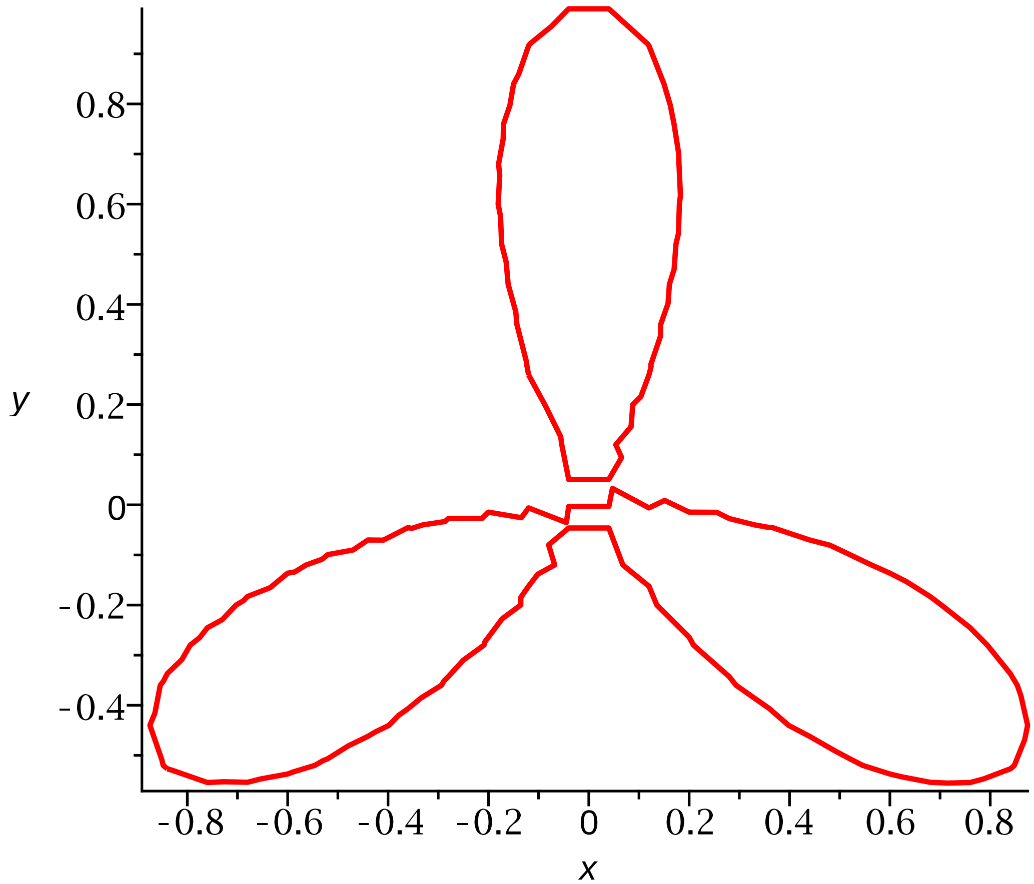

```
> implicitplot(Glg, x = -1 .. 1, y = -1 .. 1, thickness = 2,
  axes = frame, scaling= constrained, gridrefine=3);
```



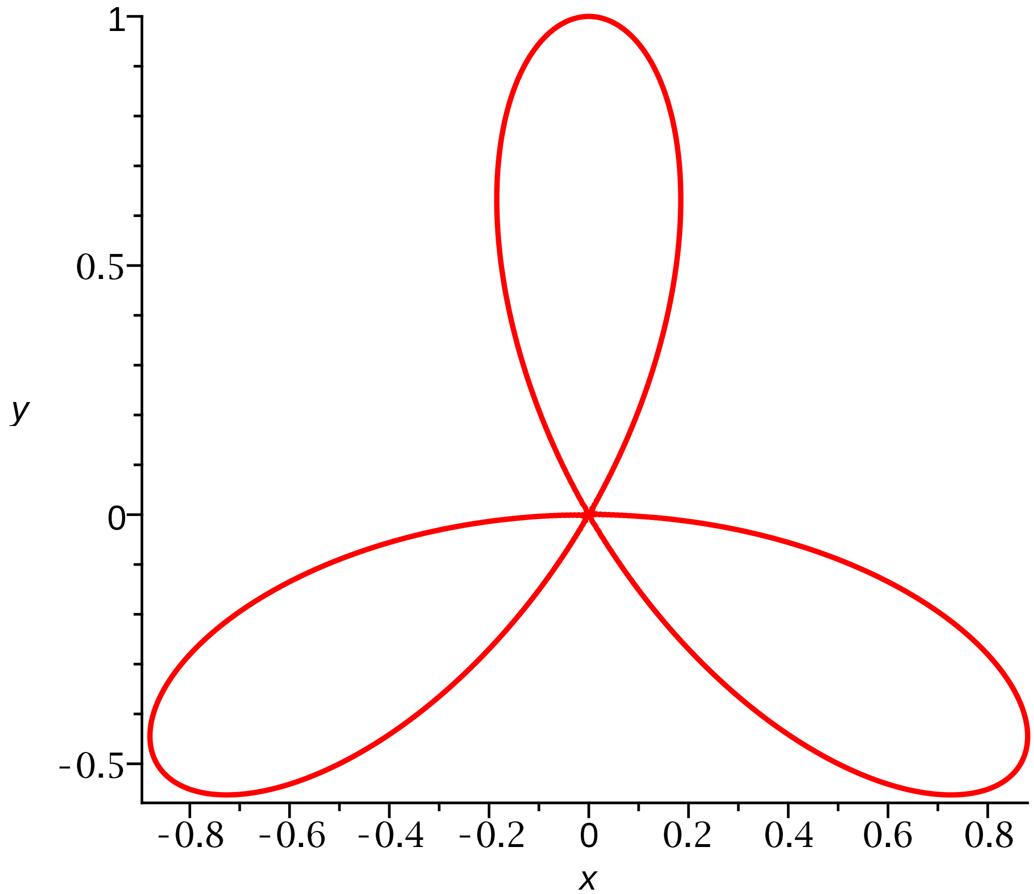

```
> Glg := (x^2 + y^2)^2 + 3*x^2*y - y^3;
```

$$Glg := \left(x^2 + y^2\right)^2 + 3\,x^2\,y - y^3 \tag{2.5}$$

```
> implicitplot(Glg, x = -1 .. 1, y = -1 .. 1, thickness = 2,
  axes = frame, scaling = constrained);
```




```
> implicitplot(Glg, x = -1 .. 1, y = -1 .. 1, thickness = 2,
  axes = frame, scaling = constrained,numpoints=60000);
```




```
> h := u*u - (v-1)*(v+1) - (w+1)*w*(w-1);
```

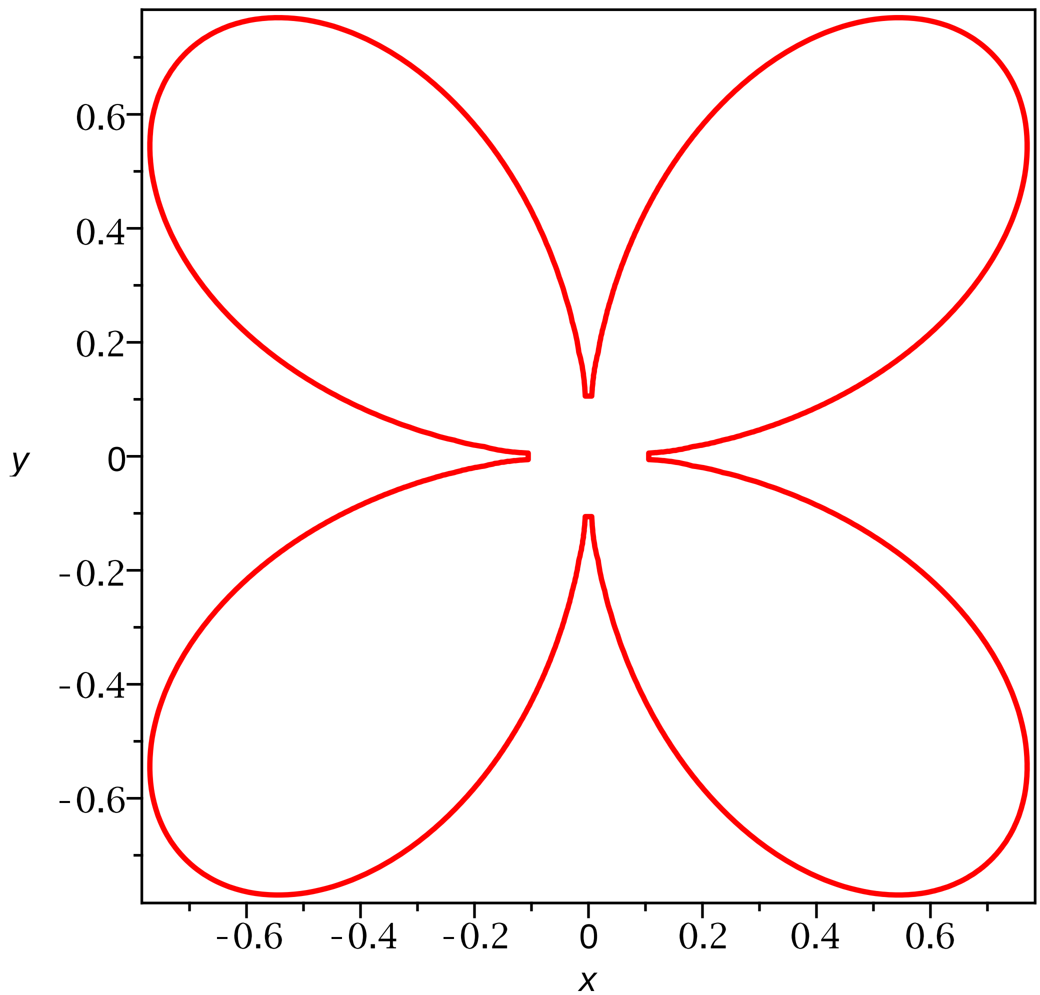
$$h := u^2 - (v-1)\,(v+1) - (w+1)\,w\,(w-1) \quad (2.6)$$

```
> implicitplot3d(h, v = -2 .. 2, u = -2 .. 2, w = -2 .. 2,
  shading = zhue, style = patchcontour, axes = boxed, numpoints
  = 10000, orientation = [45, 30]);
Error, (in iroot) powering may produce overflow
```

```
> P1 := (x^2 + y^2)^3 - 4*x^2*y^2;
```

$$P1 := \left(x^2 + y^2\right)^3 - 4\,x^2\,y^2 \quad (2.7)$$

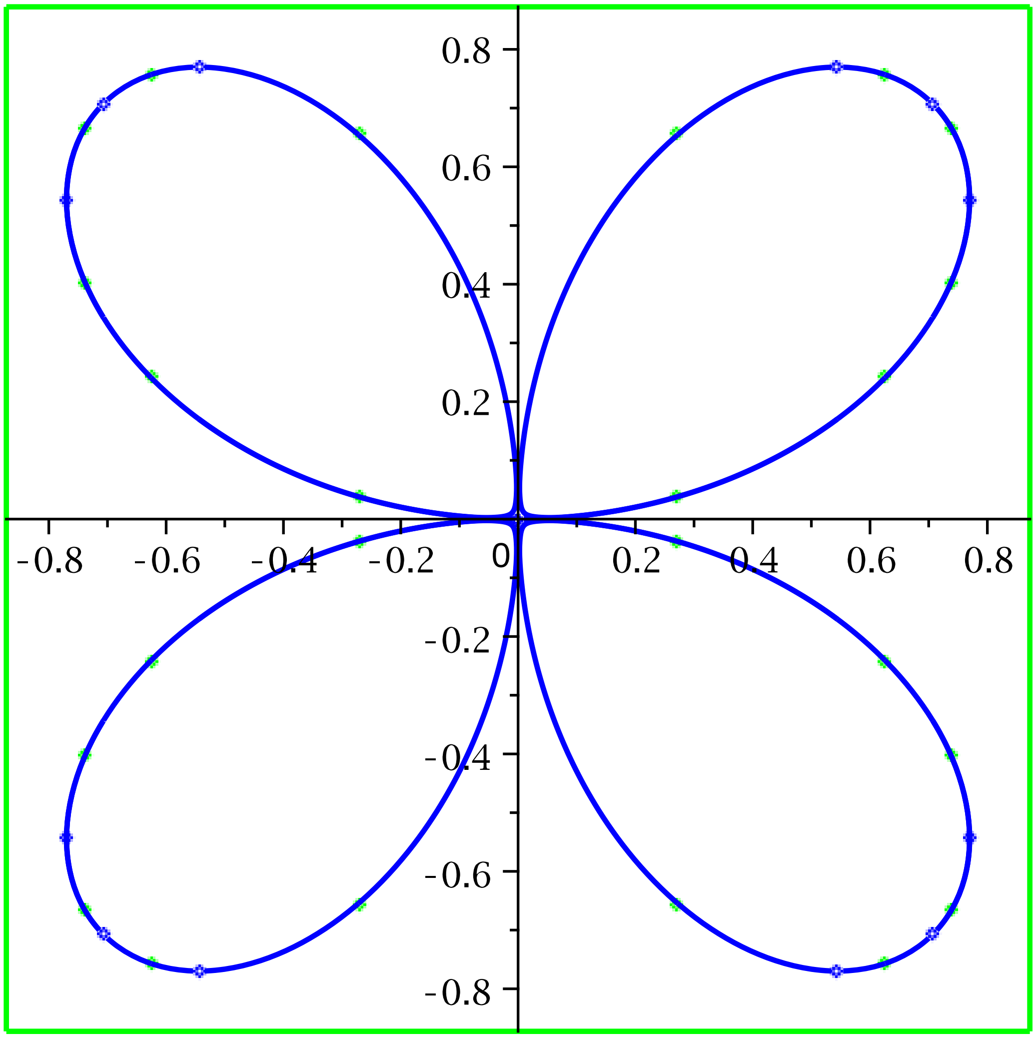
```
> implicitplot(P1, x = -.8 .. .8, y = -.8 .. .8, numpoints =
  20000, scaling = constrained, thickness = 2, axes = boxed);
```




```
> with(algcurves);
```

[*AbelMap*, *Siegel*, *Weierstrassform*, *algfun_series_sol*, *differentials*, *genus*, *homogeneous*, *homology*, *implicitize*, *integral_basis*, *is_hyperelliptic*, *j_invariant*, *monodromy*, *parametrization*, *periodmatrix*, *plot_knot*, *plot_real_curve*, *puiseux*, *singularities*] **(2.8)**

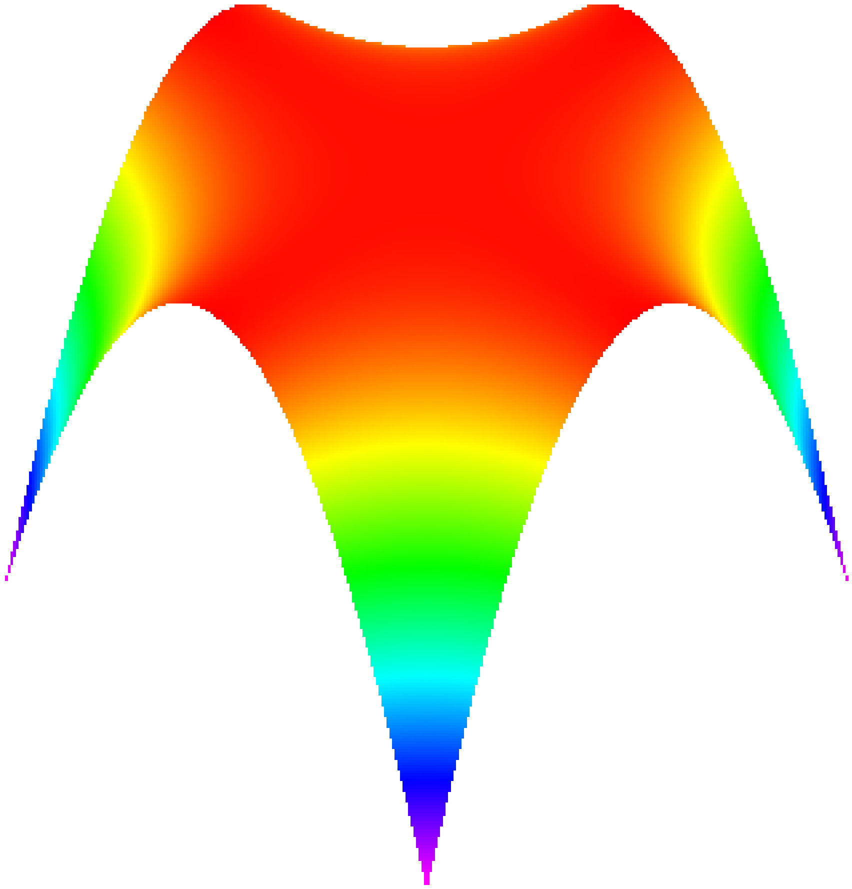
```
> plot_real_curve(P1, x , y, thickness = 2);
```




```
> plot1 := plot3d(P1, x = -.2 .. .2, y = -.2 .. .2, shading =
  zhue, numpoints = 60000, style = patchnogrid);
```

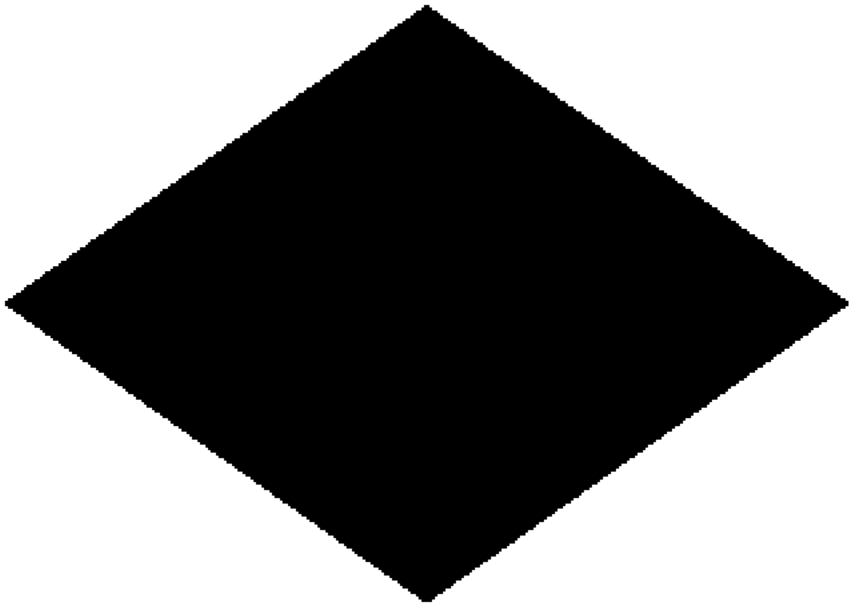
$$plot1 := PLOT3D(...) \quad (2.9)$$

```
> plot1;
```

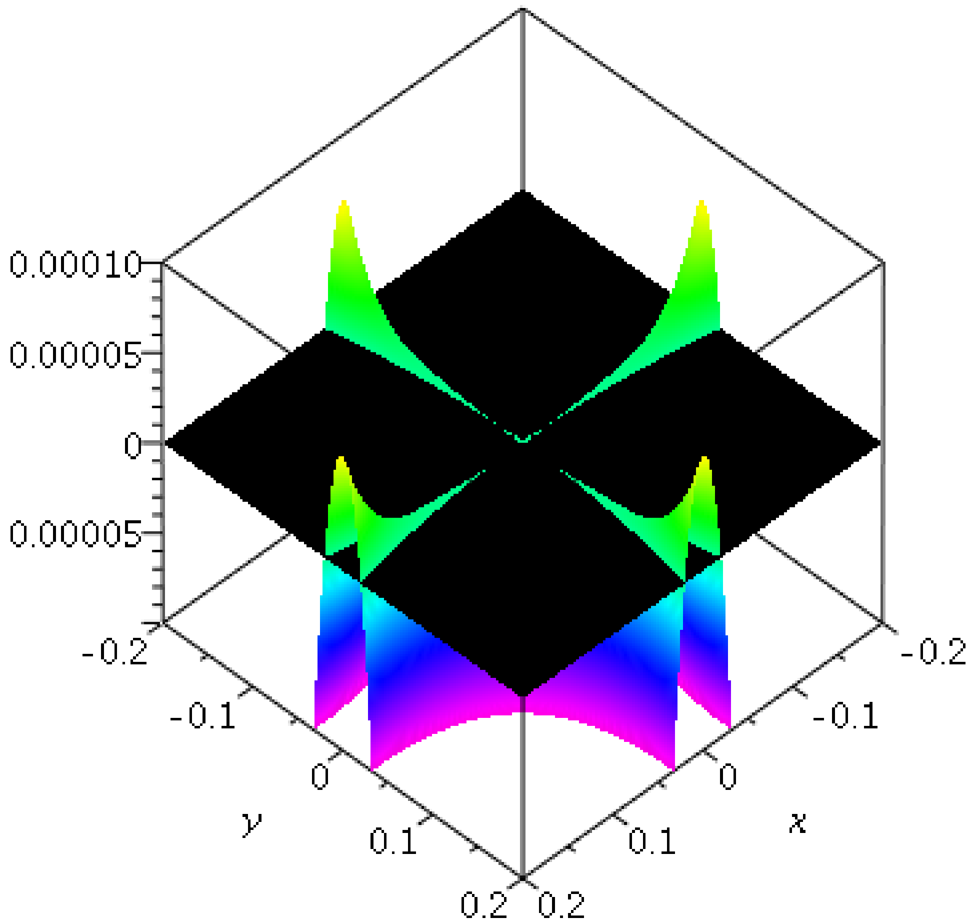


```
> plot2 := plot3d(-0, x = -.2 .. .2, y = -.2 .. .2, color =
  black, style = patchnogrid):
```

```
> plot2;
```



```
> display({plot1, plot2}, axes = boxed, view = -.0001 .. .0001)
  ;
```



Wurzeln von Polynomen

```
> restart;n := 3; a := -1/2; b := 3;
```

$$n := 3$$

$$a := -\frac{1}{2}$$

$$b := 3 \qquad \textbf{(3.1)}$$

$$> f := \frac{1}{2^n \cdot n!} (1-x)^{-a} (1+x)^{-b} \left(\frac{\partial^n}{\partial x^n} \left((1-x)^a (1+x)^b (1-x^2)^n \right) \right);$$

#Jacobi Polynom

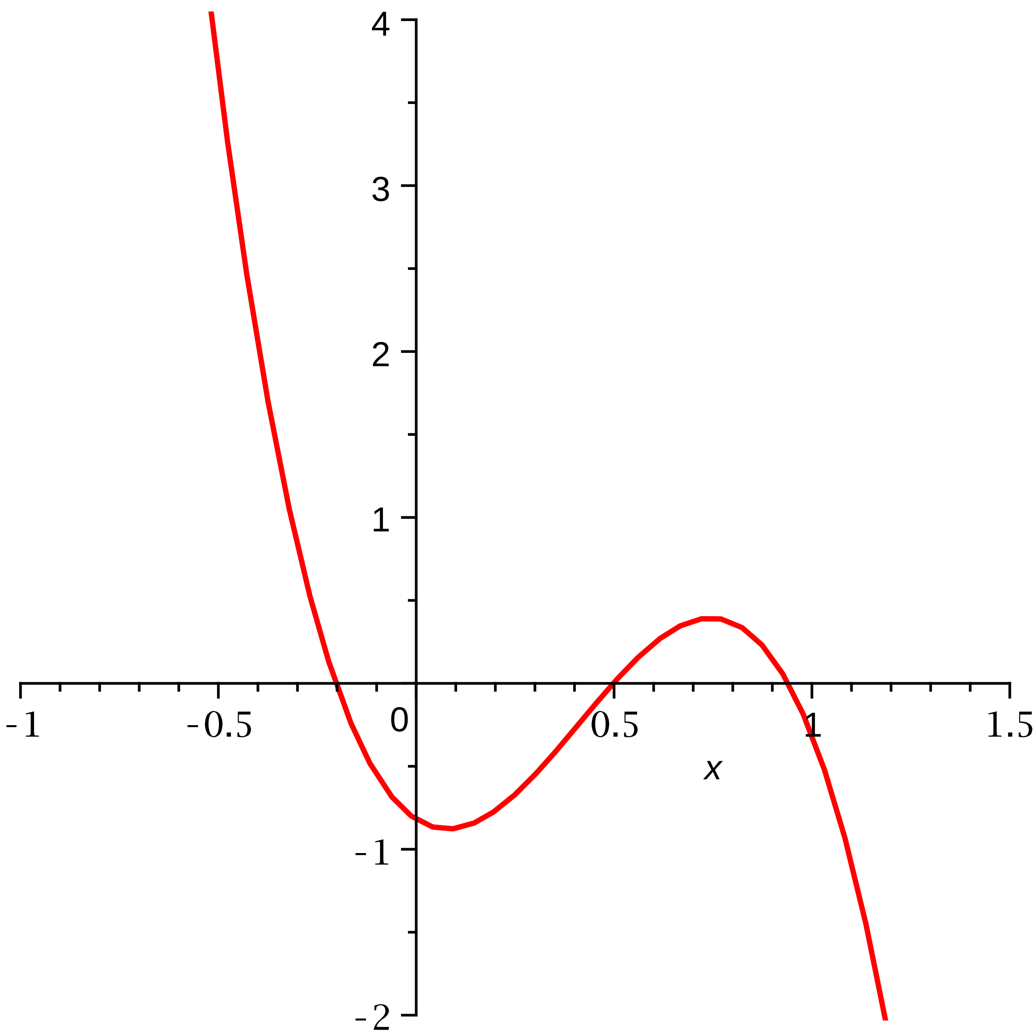
$$f := \frac{1}{48} \frac{1}{(1+x)^3} \left(\sqrt{1-x} \left(\frac{15}{8} \frac{(1+x)^3 (1-x^2)^3}{(1-x)^{7/2}} + \frac{27}{4} \frac{(1+x)^2 (1-x^2)^3}{(1-x)^{5/2}} - \frac{27}{2} \frac{(1+x)^3 (1-x^2)^2 x}{(1-x)^{5/2}} + \frac{9 (1+x) (1-x^2)^3}{(1-x)^{3/2}} \right.\right. \qquad \textbf{(3.2)}$$

$$-\frac{54\,(1+x)^2\,(1-x^2)^2\,x}{(1-x)^{3/2}}+\frac{36\,(1+x)^3\,(1-x^2)\,x^2}{(1-x)^{3/2}}-\frac{9\,(1+x)^3\,(1-x^2)^2}{(1-x)^{3/2}}$$
$$+\frac{6\,(1-x^2)^3}{\sqrt{1-x}}-\frac{108\,(1+x)\,(1-x^2)^2\,x}{\sqrt{1-x}}+\frac{216\,(1+x)^2\,(1-x^2)\,x^2}{\sqrt{1-x}}$$
$$-\frac{54\,(1+x)^2\,(1-x^2)^2}{\sqrt{1-x}}-\frac{48\,(1+x)^3\,x^3}{\sqrt{1-x}}+\frac{72\,(1+x)^3\,(1-x^2)\,x}{\sqrt{1-x}}\Bigg)\Bigg)$$

```
> simplify(%);
```

$$-\frac{1105}{128}x^3+\frac{1365}{128}x^2-\frac{195}{128}x-\frac{105}{128} \qquad (3.3)$$

```
> plot(f, x = -1 .. 1.5, -2 .. 4, thickness = 2);
```




```
> solve(f = 0);
```

$$\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}+\frac{32}{289\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}}+\frac{7}{17},\ -\frac{1}{2}\left(-\frac{896}{63869}\right. \qquad (3.4)$$

$$+\frac{128}{3757}\,I\Big)^{1/3}-\frac{16}{289\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}}+\frac{7}{17}+\frac{1}{2}\,I\sqrt{3}\,\Bigg(\Bigg($$

$$-\frac{896}{63869}+\frac{128}{3757}\,I\Big)^{1/3}-\frac{32}{289\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}}\Bigg),\ -\frac{1}{2}\,\Big($$

$$-\frac{896}{63869}+\frac{128}{3757}\,I\Big)^{1/3}-\frac{16}{289\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}}+\frac{7}{17}$$

$$-\frac{1}{2}\,I\sqrt{3}\,\left(\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}-\frac{32}{289\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}}\right)$$

```
> Lsg := [%];
```

$$Lsg:=\Bigg[\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}+\frac{32}{289\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}}+\frac{7}{17},\ -\frac{1}{2}\,\Big(\qquad \textbf{(3.5)}$$

$$-\frac{896}{63869}+\frac{128}{3757}\,I\Big)^{1/3}-\frac{16}{289\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}}+\frac{7}{17}$$

$$+\frac{1}{2}\,I\sqrt{3}\,\left(\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}-\frac{32}{289\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}}\right),$$

$$-\frac{1}{2}\,\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}-\frac{16}{289\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}}+\frac{7}{17}$$

$$-\frac{1}{2}\,I\sqrt{3}\,\left(\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}-\frac{32}{289\left(-\frac{896}{63869}+\frac{128}{3757}\,I\right)^{1/3}}\right)\Bigg]$$

```
> nops(Lsg);
```

$$3 \qquad \textbf{(3.6)}$$

```
> map(Im, Lsg);
```

$$\left[\frac{1}{4913}\,128^{1/3}\,4913^{2/3}\,2^{1/6}\sin\left(-\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{3}\,\pi\right) \qquad \textbf{(3.7)}\right.$$

$$-\frac{1}{2312}\,128^{2/3}\,4913^{1/3}\,2^{5/6}\sin\left(-\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{3}\,\pi\right),$$

$$-\frac{1}{9826}\,128^{1/3}\,4913^{2/3}\,2^{1/6}\sin\left(-\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{3}\,\pi\right)$$

$$+\frac{1}{4624}\,128^{2/3}\,4913^{1/3}\,2^{5/6}\sin\left(-\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{3}\pi\right)$$
$$+\frac{1}{2}\sqrt{3}\left(\frac{1}{4913}\,128^{1/3}\,4913^{2/3}\,2^{1/6}\sin\left(\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{6}\pi\right)\right.$$
$$\left.-\frac{1}{2312}\,128^{2/3}\,4913^{1/3}\,2^{5/6}\sin\left(\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{6}\pi\right)\right),$$
$$-\frac{1}{9826}\,128^{1/3}\,4913^{2/3}\,2^{1/6}\sin\left(-\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{3}\pi\right)$$
$$+\frac{1}{4624}\,128^{2/3}\,4913^{1/3}\,2^{5/6}\sin\left(-\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{3}\pi\right)$$
$$-\frac{1}{2}\sqrt{3}\left(\frac{1}{4913}\,128^{1/3}\,4913^{2/3}\,2^{1/6}\sin\left(\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{6}\pi\right)\right.$$
$$\left.-\frac{1}{2312}\,128^{2/3}\,4913^{1/3}\,2^{5/6}\sin\left(\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{6}\pi\right)\right)\Bigg]$$

```
> r := simplify(%);
```

Error, (in iroot) powering may produce overflow

```
> map(Re, Lsg);
```

$$\Bigg[\frac{1}{4913}\,128^{1/3}\,4913^{2/3}\,2^{1/6}\sin\left(\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{6}\pi\right) \qquad (3.8)$$
$$+\frac{1}{2312}\,128^{2/3}\,4913^{1/3}\,2^{5/6}\sin\left(\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{6}\pi\right)+\frac{7}{17},$$
$$-\frac{1}{9826}\,128^{1/3}\,4913^{2/3}\,2^{1/6}\sin\left(\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{6}\pi\right)$$
$$-\frac{1}{4624}\,128^{2/3}\,4913^{1/3}\,2^{5/6}\sin\left(\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{6}\pi\right)+\frac{7}{17}$$
$$-\frac{1}{2}\sqrt{3}\left(\frac{1}{4913}\,128^{1/3}\,4913^{2/3}\,2^{1/6}\sin\left(-\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{3}\pi\right)\right.$$
$$\left.+\frac{1}{2312}\,128^{2/3}\,4913^{1/3}\,2^{5/6}\sin\left(-\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{3}\pi\right)\right),$$
$$-\frac{1}{9826}\,128^{1/3}\,4913^{2/3}\,2^{1/6}\sin\left(\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{6}\pi\right)$$
$$-\frac{1}{4624}\,128^{2/3}\,4913^{1/3}\,2^{5/6}\sin\left(\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{6}\pi\right)+\frac{7}{17}$$
$$+\frac{1}{2}\sqrt{3}\left(\frac{1}{4913}\,128^{1/3}\,4913^{2/3}\,2^{1/6}\sin\left(-\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{3}\pi\right)\right.$$
$$\left.+\frac{1}{2312}\,128^{2/3}\,4913^{1/3}\,2^{5/6}\sin\left(-\frac{1}{3}\arctan\left(\frac{17}{7}\right)+\frac{1}{3}\pi\right)\right)\Bigg]$$

```
> simplify(%);
```

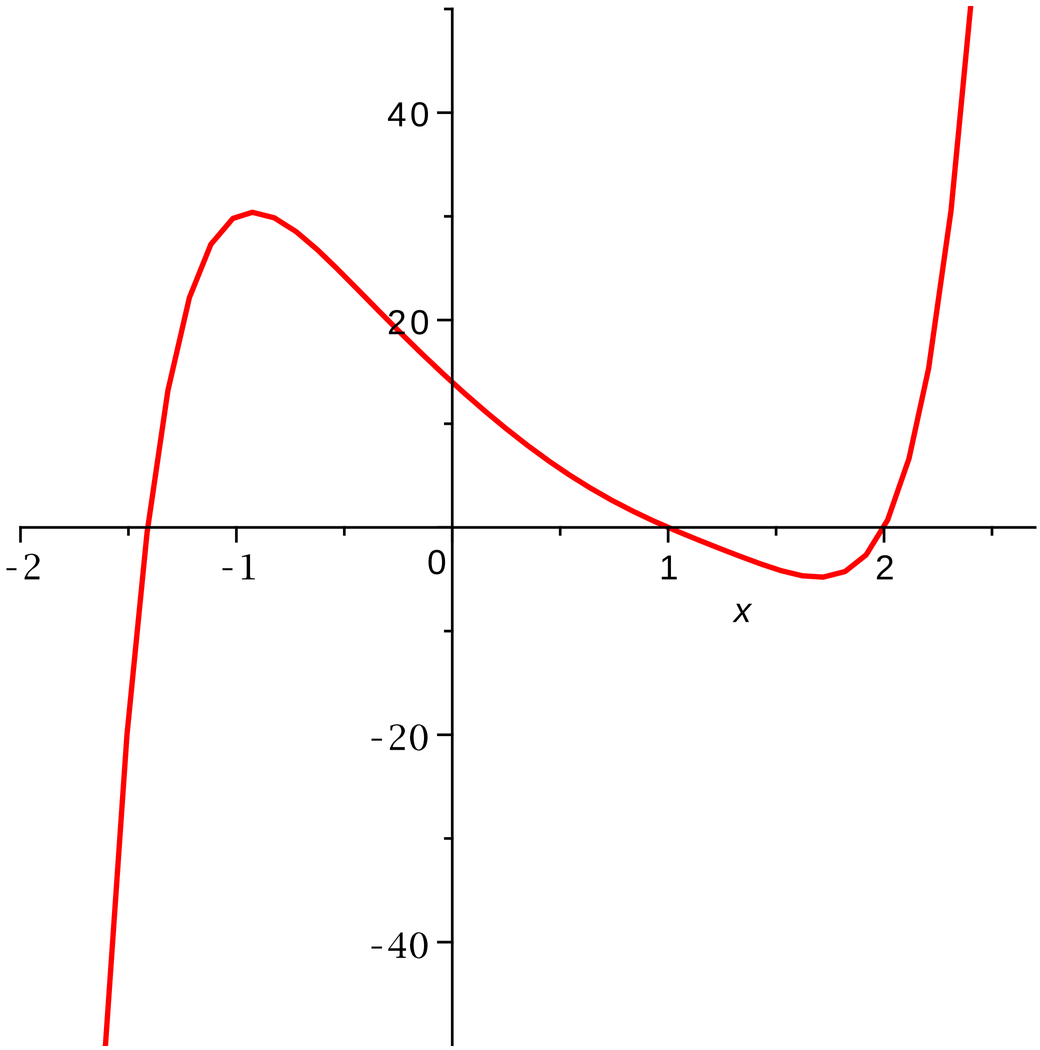
Error, (in iroot) powering may produce overflow

```
> g := x^7 - 3*x^6 + 2*x^5 + x^3 + 4*x^2 - 19*x + 14;
```

(3.9)

$$g := x^7 - 3\,x^6 + 2\,x^5 + x^3 + 4\,x^2 - 19\,x + 14 \tag{3.9}$$

```
> plot(g, x = -2 .. 2.7, -50 .. 50, thickness = 2);
```




```
> Lsg := solve(g = 0);
Error, (in iroot) powering may produce overflow
> allvalues([Lsg]);
```

$$\left[\left[\left(-\frac{896}{63869} + \frac{128}{3757}\,I\right)^{1/3} + \frac{32}{289\left(-\frac{896}{63869} + \frac{128}{3757}\,I\right)^{1/3}} + \frac{7}{17}, -\frac{1}{2}\left(\left(-\frac{896}{63869} + \frac{128}{3757}\,I\right)^{1/3} - \frac{16}{289\left(-\frac{896}{63869} + \frac{128}{3757}\,I\right)^{1/3}} + \frac{7}{17}\right.\right.\right. \tag{3.10}$$

$$+ \frac{1}{2}\,I\sqrt{3}\left(\left(-\frac{896}{63869} + \frac{128}{3757}\,I\right)^{1/3} - \frac{32}{289\left(-\frac{896}{63869} + \frac{128}{3757}\,I\right)^{1/3}}\right),$$

$$-\frac{1}{2}\left(-\frac{896}{63869}+\frac{128}{3757}\mathrm{I}\right)^{1/3}-\frac{16}{289\left(-\frac{896}{63869}+\frac{128}{3757}\mathrm{I}\right)^{1/3}}+\frac{7}{17}$$

$$-\frac{1}{2}\mathrm{I}\sqrt{3}\left(\left(-\frac{896}{63869}+\frac{128}{3757}\mathrm{I}\right)^{1/3}-\frac{32}{289\left(-\frac{896}{63869}+\frac{128}{3757}\mathrm{I}\right)^{1/3}}\right)\Big]\Big]$$

```
> fsolve(g = 0);
```

$$-1.410813851, 1., 2. \quad (3.11)$$

```
> num_Lsg := fsolve(g = 0, x, complex);
```

$$num\_Lsg := -1.410813851, -0.5084694090-1.368616488\,\mathrm{I}, -0.5084694090+1.368616488\,\mathrm{I}, 1., 1.213876335-0.9241881109\,\mathrm{I}, 1.213876335+0.9241881109\,\mathrm{I}, 2. \quad (3.12)$$

```
> for z in num_Lsg do
>   z;
> od;
```

$$-1.410813851$$
$$-0.5084694090-1.368616488\,\mathrm{I}$$
$$-0.5084694090+1.368616488\,\mathrm{I}$$
$$1.$$
$$1.213876335-0.9241881109\,\mathrm{I}$$
$$1.213876335+0.9241881109\,\mathrm{I}$$
$$2. \quad (3.13)$$

Ersetzungen

> *restart*;

```
> r := (a*x^2 + b*x + c)^3;
```

$$r := \left(a\,x^2+b\,x+c\right)^3 \quad (4.1)$$

```
> subs(a = 1, b = -1, c = 3, x = 0, r);
```

$$27 \quad (4.2)$$

```
> r;
```

$$\left(a\,x^2+b\,x+c\right)^3 \quad (4.3)$$

Bestimme den geraden Anteil von r

```
> 1/2*(r + subs(x = -x, r));
```

$$\frac{1}{2}\left(a\,x^2+b\,x+c\right)^3+\frac{1}{2}\left(a\,x^2-b\,x+c\right)^3 \quad (4.4)$$

```
> g := expand(%);
```

$$g := a^3x^6+3\,a^2x^4c+3\,a\,x^4b^2+3\,a\,x^2c^2+3\,b^2x^2c+c^3 \quad (4.5)$$

```
> cg := collect(g, x);
```

$$cg := a^3x^6+\left(3\,a\,b^2+3\,a^2c\right)x^4+\left(3\,b^2c+3\,a\,c^2\right)x^2+c^3 \quad (4.6)$$

```
> subs(x^2 = y, cg);
```

$$a^3 x^6 + (3\,a\,b^2 + 3\,a^2\,c)\,x^4 + (3\,b^2\,c + 3\,a\,c^2)\,y + c^3 \quad \textbf{(4.7)}$$

```
> algsubs(x^2 = y, cg);
```

$$3\,a^2\,y^2\,c + a^3\,y^3 + 3\,y\,b^2\,c + 3\,a\,y^2\,b^2 + c^3 + 3\,y\,a\,c^2 \quad \textbf{(4.8)}$$

```
> collect(%, y);
```

$$a^3\,y^3 + (3\,a\,b^2 + 3\,a^2\,c)\,y^2 + (3\,b^2\,c + 3\,a\,c^2)\,y + c^3 \quad \textbf{(4.9)}$$

Subs macht manchmal Fehler:

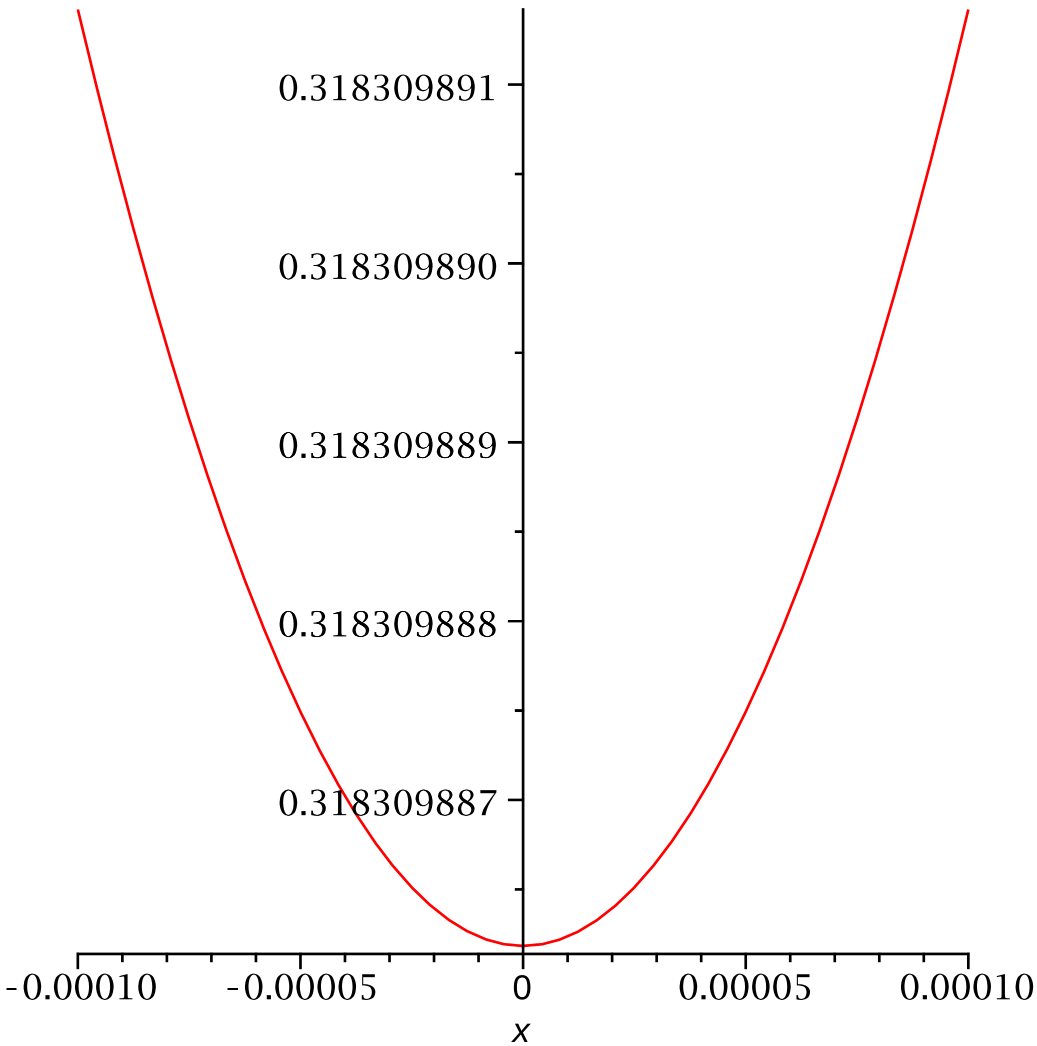
```
> h := x/sin(Pi*x);
```

$$h := \frac{x}{\sin(\pi\,x)} \quad \textbf{(4.10)}$$

```
> subs(x = 0, h);
```

$$0 \quad \textbf{(4.11)}$$

```
> plot(h, x = -0.1e-3 .. 0.1e-3);
```




```
> limit(h, x = 0);
```

(4.12)

$$\frac{1}{\pi} \tag{4.12}$$

```
> a := cos(x+y);
```

$$a := \cos(x+y) \tag{4.13}$$

```
> a = expand(a);
```

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \tag{4.14}$$

```
> b := sin(x-y):
> b = expand(b);
```

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y) \tag{4.15}$$

```
> A := cos(x)*cos(y);
```

$$A := \cos(x)\cos(y) \tag{4.16}$$

```
> A = combine(A);
```

$$\cos(x)\cos(y) = \frac{1}{2}\cos(x-y) + \frac{1}{2}\cos(x+y) \tag{4.17}$$

```
> c:= Int(sin(x),x=1..2);
```

$$c := \int_1^2 \sin(x)\,dx \tag{4.18}$$

```
> d:= Int(cos(x),x=1..2);
```

$$d := \int_1^2 \cos(x)\,dx \tag{4.19}$$

```
> combine(c+d);
```

$$\int_1^2 (\sin(x) + \cos(x))\,dx \tag{4.20}$$

```
> expand(sin(x+y));
```

$$\sin(x)\cos(y) + \cos(x)\sin(y) \tag{4.21}$$

```
> trigsubs(sin(x+y));
```

$$\left[\sin(x+y), -\sin(-x-y), 2\sin\left(\frac{1}{2}x + \frac{1}{2}y\right)\cos\left(\frac{1}{2}x + \frac{1}{2}y\right), \frac{1}{\csc(x+y)}, -\frac{1}{\csc(-x-y)}, \frac{2\tan\left(\frac{1}{2}x + \frac{1}{2}y\right)}{1 + \tan\left(\frac{1}{2}x + \frac{1}{2}y\right)^2}, -\frac{1}{2}\,I\left(e^{I(x+y)} - e^{-I(x+y)}\right)\right] \tag{4.22}$$

```
> trigsubs(sin(2*z) = 2*cos(z)*sin(z), sin(2*z)*cos(z));
```

$$2\cos(z)^2\sin(z) \tag{4.23}$$

Vereinfachungen / Annahmen

```
> restart;
> simplify(exp(x^2+ln(c*exp(y^2))-x^2));
```
$$c\,e^{y^2} \quad (5.1)$$
```
> simplify(sin(x)^2+ln(2*x)+cos(x)^2, trig);
```
$$1+\ln(2\,x) \quad (5.2)$$
```
> simplify(sqrt(x^2), assume = positive);
```
$$x \quad (5.3)$$
```
> g := int(x^2*(exp(x)+exp(-x)), x);
```
$$g := x^2\,e^x - 2\,x\,e^x + 2\,e^x - \frac{x^2}{e^x} - \frac{2\,x}{e^x} - \frac{2}{e^x} \quad (5.4)$$
```
> collect(g, exp);
```
$$\left(2+x^2-2\,x\right)e^x + \frac{-2\,x-2-x^2}{e^x} \quad (5.5)$$
```
> collect(g, x);
```
$$\left(-\frac{1}{e^x}+e^x\right)x^2 + \left(-\frac{2}{e^x}-2\,e^x\right)x - \frac{2}{e^x} + 2\,e^x \quad (5.6)$$
```
> normal((x^2-y^2)/(x+y)^2);
```
$$\frac{x-y}{x+y} \quad (5.7)$$
```
> exint := int(exp(a*t), t = 0 .. infinity);
  assume(a < 0);
  exint;
```
$$exint := \lim_{t\to\infty} \frac{e^{a\,t}-1}{a}$$
$$-\frac{1}{a\sim} \quad (5.8)$$
```
> about(a);
Originally a, renamed a~:
  is assumed to be: RealRange(-infinity,Open(0))

> additionally(a > -2);
> about(a);
Originally a, renamed a~:
  is assumed to be: RealRange(Open(-2),Open(0))

> e := ln(y/x)-ln(y)+ln(x);
```
$$e := \ln\left(\frac{y}{x}\right) - \ln(y) + \ln(x) \quad (5.9)$$
```
> simplify(e);
```
(5.10)

$$\ln\left(\frac{y}{x}\right) - \ln(y) + \ln(x) \quad (5.10)$$

```
> simplify(e) assuming y::positive;
```

$$\ln\left(\frac{1}{x}\right) + \ln(x) \quad (5.11)$$

```
> simplify(e) assuming y::positive, x::positive;
```

$$0 \quad (5.12)$$

```
> about(x);
x:
  nothing known about this object
```

Beispiel

Weitere Vereinfachungen

```
> restart;
> F := tan(x)^2 + 1;
```

$$F := \tan(x)^2 + 1 \quad (5.13)$$

```
> simplify(F);
```

$$\frac{1}{\cos(x)^2} \quad (5.14)$$

```
> convert(F, sin);
```

$$\frac{4\sin(x)^4}{\sin(2x)^2} + 1 \quad (5.15)$$

```
> convert(F,exp);
```

$$-\frac{\left(e^{I x} - e^{-I x}\right)^2}{\left(e^{I x} + e^{-I x}\right)^2} + 1 \quad (5.16)$$

```
> G := tan(3*x);
```

$$G := \tan(3x) \quad (5.17)$$

```
> G = expand(G);
```

$$\tan(3x) = \frac{3\tan(x) - \tan(x)^3}{1 - 3\tan(x)^2} \quad (5.18)$$

```
> H := tan(x) + tan(y);
```

$$H := \tan(x) + \tan(y) \quad (5.19)$$

```
> convert(H, sincos);
```

$$\frac{\sin(x)}{\cos(x)} + \frac{\sin(y)}{\cos(y)} \quad (5.20)$$

```
> normal((5.20));
```

$$\frac{\sin(x)\cos(y) + \sin(y)\cos(x)}{\cos(x)\cos(y)} \quad (5.21)$$

```
> combine((5.21));
```

$$\frac{2\sin(x+y)}{\cos(x-y)+\cos(x+y)} \quad \textbf{(5.22)}$$

```
> zaehler := numer(H1); nenner := denom(H1);
```

$$zaehler := H1$$

$$nenner := 1 \quad \textbf{(5.23)}$$

```
> H = combine(zaehler) / nenner;
```

$$\tan(x)+\tan(y) = H1 \quad \textbf{(5.24)}$$