

Numerical Methods for Data Science – Exercise Sheet 7

Exercise 18:

Let

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} -2 \\ 3 \\ 4 \\ 2 \end{pmatrix}.$$

- (a) Calculate the reduced (thin) and the full singular value decomposition $A = U\Sigma V^T$.
(b) Solve the least square problem

$$\min_{x \in \mathbb{R}^3} \|b - Ax\|.$$

- (c) State the pseudoinverse of A .

Exercise 19:

Let

$$A = \begin{pmatrix} 2 & 5 & 4 \\ 6 & 3 & 0 \\ 6 & 3 & 0 \\ 2 & 5 & 4 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 2 \\ 4 \\ 5 \\ 7 \end{pmatrix}.$$

- (a) Calculate the reduced (thin) and the full singular value decomposition $A = U\Sigma V^T$.
(b) Solve the least square problem

$$\min_{x \in \mathbb{R}^3} \|b - Ax\|$$

by writing down all solutions. Also determine the solution with the minimal norm.

- (c) State the pseudoinverse of A .

Exercise 20:

Implement the singular value decomposition and use the matrices from exercises 18 and 19 to check your implementation. You can use the algorithm from page 67 of the lecture notes. Do you obtain the same results as the existing command `np.linalg.svd`?

**Submit until July 9th 2020, 2:00 pm in the ILIAS.
Review in the exercise course on July 10th 2020.**