Heinrich Heine Universität Düsseldorf

MATHEMATICAL INSTITUTE

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NAME: ____________________

MAT-NR.: ____________________

Numerical Methods for Data Science – Exercise Sheet 6

Exercise 14:

Consider the matrix

$$A = \begin{pmatrix} -2 & -2 & -2 & -2 \\ -2 & -1 & -1 & -1 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Calculate the QR decomposition of A using

(a) the Gram-Schmidt-Algorithm,

(b) Householder reflections.

Use it to find the solution to $Ax = b$ where $b = (-20, -11, -2, 4)^T$.

Exercise 15:

Consider again the matrix from exercise 14. Perform one step of the QR algorithm. If you weren't able to calculate the QR decomposition, you can also use the computer here. After that, compare the approximations that you get for the eigenvalues to the real eigenvalues of A. You can use the computer to calculate these as well. How do the eigenvalues compare to the approximation? What structure of A do you expect in the limit when taking many steps of the QR algorithm?

Exercise 16:

Let

$$A = \begin{pmatrix} 1 & -2 & 3.5 \\ 1 & 3 & -0.5 \\ 1 & 3 & 2.5 \\ 1 & -2 & 0.5 \end{pmatrix}.$$

Perform step 1 of the SVD algorithm that transforms A into a bidiagonal matrix B as explained in the lecture.

Exercise 17:

Let B be a bidiagonal matrix And $H = \begin{pmatrix} 0 & B^T \\ B & 0 \end{pmatrix}$. Let P be a permutation matrix of the form $P = (e_1|e_{n+1}|e_2|e_{n+2}|\dots|e_n|e_{2n})$, where e_i is the i-th unit vector. Show that $P^T HP$ is a symmetric tridiagonal matrix with zeros on the diagonal. Also provide the exact form of the superdiagonal.

Submit until July 2nd 2020, 2:00 pm in the ILIAS.
Review in the exercise course on July 3rd 2020.