

Numerical Methods for Data Science – Exercise Sheet 5

Exercise 12:

Let A be a matrix, x be a vector with $\|x\|_2 = 1$ and λ be a scalar. Define $r = Ax - \lambda x$. Show that there exists a matrix E such that $A + E$ has eigenvalue λ with corresponding eigenvector x and $\|E\|_F = \|r\|_2$, where $\|\cdot\|_F$ denotes the Frobenius matrix norm.

Exercise 13:

Implement the Power Method, the Inverse Iteration as well as the Rayleigh Quotient Iteration.

Test your implementations on the matrix $T_n + \epsilon I \in \mathbb{R}^{10 \times 10}$, where T_n is the matrix from chapter 1, $|\epsilon| \ll 1$ and I is the identity matrix, by always searching for the largest eigenvalue in magnitude. Plot the error after each iteration of all methods in a combined plot. How does the convergence speed depend on ϵ for the different methods? Always start with the same initial vector $v^{(0)}$ for all methods which is a good guess for the eigenvector that belongs to the largest eigenvalues in magnitude. For the Inverse Iteration, choose μ to be a good guess on the largest eigenvalue in magnitude as well.

**Submit until June 18th 2020, 2:00 pm in the ILIAS.
Review in the exercise course on June 19th 2020.**