Heinrich Heine Universität Düsseldorf

MATHEMATICAL INSTITUTE

PROF. DR. CHRISTIANE HELZEL
DAVID KERKMANN

MAY 7TH 2020

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NAME: ____________________

MAT-NR.: ____________________

Numerical Methods for Data Science – Exercise Sheet 2

Exercise 5: Let $A \in \mathbb{R}^{n\times n}$ be symmetric and positive definite and $b \in \mathbb{R}^n$. Let $x \in \mathbb{R}^n$ be the true solution to $Ax = b$ and $r_k = b - Ax_k$ be the residual for iteration k.

Let $f_X : \mathbb{R}^n \to \mathbb{R}^{\geq 0}, r \mapsto \|r\|_X := (r^T X r)^{1/2}$ for any symmetric positive definite matrix X.

Show that f_A and $f_{A^{-1}}$ define vector norms.

Prove that $\|r_k\|_{A^{-1}} = \|x_k - x\|_A$.

Exercise 6: Prove the following:

Let A be symmetric positive definite and $Q^{n\times k}$ have full column rank. Then $T = Q^T AQ$ is also symmetric positive definite.

Exercise 7: Let $A = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$.

Compute the companion matrix C by manually computing K as defined in the lecture and then solving for the last column of C. Also compute the $K = QR$ decomposition and check the upper Hessenberg form of $Q^T AQ$ (you can do this with a program).

Exercise 8: **Programming exercise**

Implement the Arnoldi algorithm using Python by writing a function that receives a matrix A and a vector b and returns the resulting matrices Q and H.

Test you problem with a random matrix $A \in \mathbb{R}^{10\times 10}$ and a random vector $b \in \mathbb{R}^{10}$ and confirm the desired properties of Q and H.

Submit until May 14th 2020, 2:00 pm in the ILIAS.
Review in the exercise course on May 15th 2020.