

Numerical Methods for Data Science – Exercise Sheet 1

Exercise 1: Prove Lemma 1.1.

Exercise 2: Consider the matrix T_n from section 1.1.1. We want to apply Jacobi's method on the linear system $T_n x = b$. Derive the iteration matrix G and calculate its 1-norm and ∞ -norm. Does the method converge to the correct solution for any right hand side b and any starting vector x_0 ?

Exercise 3: Prove Theorem 1.5. You can work out the proof in J. Demmel's book, page 280.

Exercise 4: **Programming exercise**

Choose one between Jacobi's method, Gauss-Seidel and SOR.

- (a) Implement the chosen algorithm using Python by writing a function that receives the left hand side A , the right hand side b of a linear system of equations $Ax = b$ and a tolerance as input parameters and returns the resulting iteration once the change $\|x_{k+1} - x_k\|_\infty$ is smaller than the given tolerance. If you choose the SOR method, you will need to add an additional input parameter w , the relaxation parameter.
- (b) Change your function so that it returns all iterations. Plot the error against the number of iterations similar to as was done in the lecture. Compare the plots for a random matrix $A \in \mathbb{R}^{1600 \times 1600}$ and the matrix T_{1600} that arises from the discretisation of the one-dimensional Poisson problem. If you use the SOR method, you can chose w . State your observations.

**Submit until May 7th 2020, 2:00 pm in the ILIAS.
Review in the exercise course on May 8th 2020.**